5-2011

Exploring the Relationship Between Orpda and Teachers' Conceptual Understanding of Place Value

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I am submitting herewith a dissertation written by Jamie Howard Price entitled "Exploring the Relationship Between Orpda and Teachers' Conceptual Understanding of Place Value." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Education.

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Exploring the Relationship Between Orpda and Teachers' Conceptual Understanding of Place Value

A Dissertation
Presented for the Degree of
Doctor of Philosophy
The University of Tennessee, Knoxville

Jamie Howard Price
May 2011

Dedication

This dissertation is dedicated to my wonderful and loving husband, Casey Price. I cannot even begin to tell you how much I appreciate your love, guidance, and support throughout this journey. Throughout my roller coaster of emotions, you have been there to celebrate the highs and comfort me during the times of stress. You have encouraged me to take another step forward each day and realize my goal. Thank you for taking each step with me. I love you!

Acknowledgements

There are not enough words to express thanks to all of the people who have helped me throughout this journey. I have been so blessed to have the unending support of my family, friends, teachers, and colleagues, and each one of you has made this experience into what it has become—the realization of a goal I set for myself many years ago. In particular, there are a few specific people that I would like to especially thank.

I would first like to thank each of my committee members – Dr. Colleen Gilrane, Dr. Jo Ann Cady, Dr. Amy Broemmel, and Dr. Charles Collins. I could not have put together a more supportive and encouraging committee and I am so truly thankful for all of you. To my committee chair, Dr. Gilrane, your guidance and support have been invaluable throughout this entire journey. I have appreciated your encouraging words and smiles, both in person and via email ☺. You have helped me realize the beauty of doing research and touching the lives of others. To Dr. Cady, thank you for "taking me under your wing" from early on into this journey. You have not only been my adviser, but also a friend that I can turn to for honest support and inspiration. Thank you for filling this experience with laughs along the way! To Dr. Broemmel and Dr. Collins, I cannot tell you how much I appreciate your willingness to take time out of your schedules to support me along this journey. Thank you for taking the time to read through my dissertation and offer valuable feedback so that I can be proud of this project.

I would also like to take a moment to thank all of my family members. You have been there for every moment along the way. I appreciate you listening to me stress over whether or not I could actually do this and encouraging me not to give up on my goal. Thank you for always being a constant support to me and continuing to inspire me to reach even beyond my potential. I love all of you very much!

Finally, thank you to all of the teachers that participated in this project. I appreciate your open and honest thoughts and reflections. I am so glad to have had the opportunity to meet you and I know that many lives will be touched by your teaching.

Abstract

The purpose of this case study was to understand whether or not the use of an invented number system, called Orpda, helped teachers develop a deeper understanding of place value in hopes that this will translate into their own teaching of place value concepts. Thirteen teachers enrolled in a graduate mathematics education course served as the participants for this study. Data were collected from teachers' reflections on various activities related to Orpda, pre- and post-Orpda concept maps teachers created, online discussions between the teachers, teacher demographic sheets, and an interview with the instructor of the course.

Analysis of the teachers' reflections revealed that Orpda increased teachers' attention to three critical components necessary for developing a conceptual understanding of place value, namely unitizing, regrouping, and recognizing the meaning of different place values within a multi-digit number. In addition, Orpda encouraged teachers to reflect on their own teaching of place value. Comparing the structures of the teachers' pre-Orpda and post-Orpda concept maps showed changes in some cases but did not reveal clear patterns. Analysis of the categories teachers included in pre- and post-Orpda maps revealed that teachers were moving from a procedural to a more conceptual view of place value, as did the analysis of squared adjacency matrices created from each teacher's pre- and post-Orpda concept maps.

Four conclusions can be drawn from this study: (a) Orpda increased teachers' attention to the importance of unitizing in place value, (b) Orpda encouraged teachers to reflect deeply on their thinking, (c) Concept maps show promise for revealing and documenting changes in conceptual understanding, and (d) Orpda increased teachers' attention to the importance of patterns in understanding place value. Further research is needed using Orpda with different groups and numbers of teachers, and in different settings, e.g., longer full semesters and teacher

professional development meetings. Research exploring the use of follow-up interviews to accompany concept maps and enhance the assessment of conceptual understanding is also recommended. This study indicates two recommendations for practice in teacher education, the importance of a classroom environment that supports reflection, and the careful choosing of activities to provide appropriate challenge.

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Chapter I

Introduction

The mathematical concepts introduced in the elementary curriculum form the foundation for the remaining mathematics that students will learn as they progress through high school and beyond. The place value system is taught early in the elementary mathematics curriculum, as it provides a building block for all future learning related to mathematics. However, place value should not be considered as elementary since it involves many deep and abstract concepts that are not naturally understood by the learner. As a result, a heavy burden is placed upon elementary teachers to realize the complexities surrounding a complete understanding of place value. In order to help their students gain a conceptual understanding of place value, teachers must possess a conceptual understanding of place value themselves (Ball, 1993).

Within many of the science, technology, engineering, and mathematics (STEM) fields, the dichotomy between procedural and conceptual knowledge is present and must be understood as both forms of understanding are essential. Many classrooms in the United States focus on developing children's understanding of the procedures used to solve problems (Stigler & Hiebert, 1999). Recent reform efforts, on the other hand, encourage a shift towards expanding students' conceptual understanding in order to fully understand mathematical ideas (Comiti & Ball, 1996; NCTM, 2000). Meeting these demands, however, requires teachers to possess a "profound understanding" of mathematics, as defined by Ma (1999):

> [A] profound understanding of fundamental mathematics goes beyond being able to compute correctly and to give a rationale for computational algorithms. A teacher with profound understanding of fundamental mathematics is not only aware of the

> conceptual structure and basic attitudes of mathematics inherent in elementary mathematics, but is able to teach them to students. (p. xxiv)

One of the major challenges that teacher educators face is ensuring that preservice teachers are equipped to teach students using methods outlined by these reform efforts. This often requires teacher educators to find ways to help these teachers reconceptualize their understanding of the previously learned mathematical ideas. Since many teachers are familiar with the base-ten number system, they struggle to fully comprehend the difficulties their students have when trying to understand the abstract concept of place value. As a result, Dr. Theresa Hopkins and Dr. Jo Ann Cady (2007) developed a new number system which they call Orpda. This number system utilizes symbols rather than numerals to represent particular values as indicated in Table 1.1. The purpose of creating Orpda was to remove teachers from their familiar environment of working in the base-ten number system and force them to conceptualize the ideas related to place value in hopes that their experiences with Orpda would be similar to their students' experiences with learning place value in the base-ten number system (Hopkins & Cady, 2007).

Table 1.1
Number of Objects and Representative Symbol and Name in the Orpda Number System

No objects	~	tilde
1 object	*	star
2 objects	@	at
3 objects	#	pound
4 objects	^	carat

"What is the value of @*#?" by T.M. Hopkins and J.A. Cady, 2007, *Teaching Children Mathematics*, *13*, p. 435.

Purpose of the Study

The purpose of this study is to understand the relationship between Orpda and teachers' understanding about place value. Specifically, the study will consider how teachers' conceptual understanding about place value develops throughout their experiences with Orpda.

Research Question

The study will be guided by the following research question:

How does Orpda relate to teachers' conceptual understanding of place value?

Specifically, the study will attempt to answer the following three sub-questions:

1. What do the teachers' reflections on the instructional tasks done in Orpda reveal about their understanding of place value?
2. What do comparisons of pre- and post-Orpda concept maps reveal about teachers' understanding of place value concepts after working with Orpda?
3. What connections do teachers make between Orpda and the Arabic number system?

Significance

This study attempts to examine the relationship between Orpda and teachers' understanding of place value. Teachers already possess familiarity with the base-ten number system and how it is used to perform mathematical calculations. As a result, many teachers hold the belief that the concepts surrounding place value can be learned quickly and easily by their students, especially in the early elementary grades, and therefore do not provide the necessary time to allow these ideas to fully develop (Harvin, 1984) . Orpda was created with the hope that it would force reflection, lessen the temptation to convert to base 10, and force teachers to revisit their own place value understanding. In doing so, the deep understanding required to be successful at teaching place value to children may be developed, and teachers may realize how complex place value systems are and how much time is required to develop the associated concepts in their elementary students. In addition, they may realize what constitutes a conceptual understanding of place value and develop ways to better assess their students' knowledge of these fundamental ideas.

In addition to exploring a method of deepening teachers' understanding about place value, this study also contributes to a line of research related to promoting reform-oriented initiatives in mathematics classrooms. Much of the literature surrounding the reform movement in mathematics education has focused on the benefits that it provides for students, giving specific accounts of ways teachers have implemented new instructional methods in their classrooms (Ball, 1993; Cady, 2006; Van de Walle, 2007). This study, however, focuses on one way to help teacher education programs understand ways to work with teachers to deepen their understanding about topics for which they are already familiar, such as place value, while also encouraging the use of reform approaches to teaching mathematics. Since the activities that they will do in Orpda can easily be translated to work with the base-ten number system, this study will allow teachers to become students in a reform-oriented classroom in order to experience these new instructional techniques for themselves and better understand how to implement this way of teaching in their own classrooms.

Limitations

Some conditions that I did not have any control over may have influenced the results of this study. First, since the mathematics education course that was used for this study is considered to be an elective, I did not have any control over the individuals that signed up for the class. Therefore, I was not able to control the size of the participant pool for this study, and any individuals choosing not to participate were excluded. Furthermore, I was not able to randomly select the teachers to be part of the sample since they were enrolling in the course to fulfill requirements within their respective degree programs.

Second, research regarding conceptual understanding indicates that the process is both gradual and time-consuming (Ozdemir & Clark, 2007). Since the length of time for summer courses is established by the university at which the study will be conducted, I was unable to control the length of time that the participants had to allow for the development of a conceptual understanding of place value. While a longitudinal study in this area of interest would be helpful, it is beyond the scope of the requirements for this study. I attempted to account for some additional time needed for the development of conceptual understanding by having the teachers construct a second concept map at the end of the course rather than immediately following their work with Orpda.

Delimitations

The results of this study may have also been influenced by factors that I, as the researcher, did choose to control. I chose to structure this study as a case study in order to gain an in-depth understanding of the relationship between Orpda and the teachers' place value understanding. I recognize that using a case study approach may limit the generalizability of the results from this study, but I felt that focusing on a smaller number of participants would allow me to collect the data needed to answer the research questions. Future research can be built from this study to work with a larger number of participants in order to further the research base associated with Orpda.

Assumptions

This study was guided by a few assumptions. This study assumed that the teachers brought some working knowledge of place value with them to the study and therefore were asked to illustrate that knowledge through a concept map before being introduced to Orpda. In addition, the researcher assumed that the participants took the process of constructing the concept map seriously both before and after Orpda, along with taking the time for necessary reflection, so that the results of this study accurately depicted any possible relationships between their experiences with Orpda and their understanding of the concepts related to place value.

Summary and Organization of the Study

Preparing teachers to teach elementary mathematics topics is a challenging task as their familiarity with the topics can mask a lack of deep, conceptual understanding (Ball, 1988). Consequently, opportunities must arise within the teacher education program for teachers to expose their true understanding and recognize the need for a strong knowledge base in order to effectively teach these topics. Orpda provides one way for teachers to reconceptualize their understanding of place value. The next chapter will outline the key components associated with a conceptual understanding of place value as well as present research related to conceptual understanding and its assessment. It will conclude with a description of the theoretical framework guiding this study. Chapter three will present the methodology associated with the research, outlining the design of the study, methods used to collect data, and a description of how the data was analyzed. Ways to establish trustworthiness and credibility for the study along with ethical considerations will also be presented. In chapter 4, the results of the data analysis will be presented as they relate to each of the three sub-questions, while chapter 5 will provide the conclusions that can be drawn from the research. The final chapter will conclude with recommendations for further research and practice.

Definitions of Terms

For the purpose of this study, the following terms will be defined: symbols, numeration system, base, place-value numeration, unitizing, preservice teacher, in-service teacher, teacher, concept map, procedural understanding, and conceptual understanding.

Symbols are letters, figures, or other characters used to designate something.

A *numeration system* consists of two parts:

1) a set of fundamental symbols used to denote the size of fundamental sets
2) an understood arrangement of these symbols to measure sets that are not one of the fundamental-sized sets

For example, within the base 10 numeration system, each of the fundamental symbols 0,1,2,3,4,5,6,7,8, and 9 denote a fundamental set. We, then, use combinations of these symbols to denote the sizes of other sets. Within Orpda, the fundamental symbols used are ~, *, @, #, and ^.

The *base* of a numeration system is the number of symbols used in a numeration system to represent the sizes of the basic sets. Since our number system uses ten basic symbols, it is therefore referred to as a base 10 numeration system, while Orpda is considered a base 5 numeration system. In addition, the base of a numeration system determines the grouping size that each place value represents. For example, within the base 10 number system, 10 individual units create one group of ten, and ten groups of ten create one group of one hundred. In Orpda, the number 5 is referred to as a flub, and therefore flub individual units create one group of flub while flub flubs create a skoobrat.

The major principle of *place-value numeration* refers to the idea that the positions of digits in numbers determine what they represent or which size group they count.

The act of *unitizing* involves combining multiple objects together to form one group. Within the base 10 numeration system, it is important that children are able to unitize groups of ten objects; that is, they are able to recognize that 10 ones can be combined to form one group of ten, while 10 tens can be combined to form one group of one hundred, and so on.

Preservice teachers are students enrolled in a four or five year teacher education program with an end result of becoming a licensed teacher.

In-service teachers are current classroom teachers.

In this study, the term *teacher* will refer to both preservice teachers and in-service teachers, with the viewpoint that from the time an individual enrolls in a teacher education program and throughout his or her teaching career, he or she is placed on a continuum of the teaching profession, while the term *instructor* will refer to the person presenting the Orpda ideas and activities.

A *concept map* is a visual representation of the mental connections that an individual has made related to a given topic.

A *procedural understanding of mathematics* involves an algorithmic understanding of the subject. Individuals with a procedural understanding of mathematics are able to follow an algorithm or procedure correctly to arrive at the answer to a computational problem.

A *conceptual understanding of mathematics* involves an understanding of the interrelations between pieces of knowledge in mathematics. This knowledge is rich in connections with many pathways connecting ideas and concepts. Therefore, individuals with a conceptual understanding of mathematics can apply and adapt ideas previously learned to new situations, convert easily between multiple representations, and associate meaning to their results from a computational problem.

Chapter II

Review of the Literature

Past research indicates that before entering school, many children have already developed an informal sense of number and quantity (Briars & Siegler, 1984; Carpenter, Fennema, Franke, Levi, & Empson, 1999; Schaefer, Eggleston, & Scott, 1974). During the early elementary years, teachers must recognize the knowledge that children bring with them to the classroom and work to deepen their number sense over a broad curriculum, ranging from basic counting techniques to a complete understanding of place value. This presents a challenging task for teachers and in order to meet these demands, they must possess a strong knowledge of place value and number sense for themselves. This knowledge is lacking in elementary teachers (Ma, 1999). Orpda was created to encourage reflection on place value concepts and increase teachers' conceptual understanding and pedagogical content knowledge.

Therefore, in order to understand how Orpda relates to teachers' conceptual understanding about place value, the principles that govern a conceptual understanding about place value must first be understood. Jones et. al. (1996) identified four key components that are central to developing a conceptual understanding of place value, namely counting, unitizing, regrouping, and number relationships. Consequently, Orpda activities were created to develop these concepts in teachers since teachers are already familiar with the base-ten number system. While working with Orpda, the teachers will be theoretically transported back to elementary school as they must learn how to work with this new number system from the beginning stages. Consequently, research related to children's knowledge about counting, unitizing, regrouping, and number relationships will serve as a starting point to understanding teachers' knowledge about place value within the Orpda numeration system.

This chapter will begin by reviewing the mathematics education research related to the four key components of understanding place value, focusing on both children's understanding as well as that of teachers. Next it will consider ways to assess conceptual understanding, focusing on the one that will be utilized within this study. The chapter will conclude with a discussion of the theoretical framework that will be used to guide the research.

Key Components of Understanding Place Value

Counting

Several researchers have focused on the importance of counting as the first stage of developing place value concepts (Bell, 1990; Kamii & DeClark, 1985; Steffee, Cobb, & von Glasersfeld, 1988; Van de Walle, 2007). While the process of counting may seem trivial, especially to teachers who have been counting for much of their lives, teaching children how to count and understand what they are doing is much more difficult. Schwartz (2008) identifies nine elements involved in counting a group of objects. These elements are:

1. Locate a collection of objects on which to act.
2. Understand that there is such a thing as quantity.
3. Utter a correct sequence of words.
4. Identify a matching object from the collection for each word uttered.
5. Omit no objects in the collection.
6. Identify no object more than once.
7. Stop uttering words when each item in the collection has been identified.
8. Declare that the last word uttered is the "count" of the characteristics of the objects themselves.

9. Understand at the end of the process that the last word uttered establishes a characteristic of the collection rather than a characteristic of any specific object.

The first stage in being able to count correctly in the base-ten numeration system involves learning the language and understanding the correct sequence used to utter the word names. Seeking out appropriate language patterns among the word names is helpful for children to begin understanding how to count. However, the English language presents several inconsistencies that make finding a pattern difficult. For example, the quantity ten is represented using three different names, namely *ten*, *-teen* as in thirteen, and *–ty* as in thirty. Second, the word names eleven and twelve are arbitrary and do not follow any language pattern making them difficult to remember. Third, the number names for the teens reverse the order of the tens and ones which further confuses children as they begin to understand place value. For instance, the word name *fourteen* combines *four* to represent four ones along with *–teen* to represent one group of ten.

Researchers have studied the English language in contrast with the Chinese language to further understand the achievement gap related to children's counting abilities (Cotter, 2000; Ho & Fuson, 1998). Ho and Fuson suggest that the English language reinforces an embedded-decade cardinal understanding as the word names for numbers up to 100 are built on decades. On the other hand, the Chinese language counts the numbers beyond *ten* as *ten 1*, *ten 2*, *ten 3*, and so forth. This pattern continues for higher numbers such as learning the word name of 30 as *3 ten*, followed by 31 as *3 ten 1*. They argue that the embedded-ten cardinal understanding that the Chinese language reinforces allows Chinese children to be able to count to higher numbers at an earlier age than American children. Cotter strengthens the argument made by Ho and Fuson recognizing that in order to count to 100, American children must learn a total of twenty-eight words, including the word names for 1 through 19, the decade names for 20 through 90, as well as the word name for 100. In contrast, Chinese children only have to learn a total of eleven words, namely the words for 1 through 10 as well as the word for 100.

After learning the language of counting, place value concepts can begin to be established through counting techniques as children start to understand the relationship between the word names and their representation of quantity. Thompson (1990) describes three ways in which children count sets which also provide ways for them to think about quantity. These three ways include counting by ones, counting by groups and singles, and counting by tens and ones. Most children begin associating the number of objects in a set with the idea of quantity using a counting by ones approach. While this is not the most efficient nor does it begin to develop base-ten concepts, it does help to reinforce the idea of quantity. Next, children use a counting by groups and singles approach. Children that use this approach to counting a set of 32 objects would do so by saying, “One, two, three bunches of ten, and one, two singles.” Clearly, this technique allows children to begin unitizing as they group by ten and count each group as a single item, but this technique does not end by telling how many items there are and therefore should be combined with a counting by ones approach to establish quantity. Finally, a counting by tens and ones approach is the common way that most adults count objects. A set of 32 objects counted in this way would be done by saying, “Ten, twenty, thirty, thirty-one, thirty-two.” This method does encourage the understanding of quantity, but it does not enforce the concept of unitizing as explicitly as the previous technique. Thompson encourages children to have opportunities to count objects in many different ways in order for them to make meaning of quantity as well as develop an understanding of grouping by tens.

Unitizing

The process of unitizing involves being able to combine multiple objects together to form one group. Unitizing is one of the major ideas surrounding base-ten numeration as children learn that 10 individual ones can be combined to form one group of ten and this group can be perceived as a single entity. This is a major shift in thinking from having previously focused on individual objects. Consequently, while unitizing is a key component of understanding place value, it does not follow naturally and must be developed.

Cobb and Wheatley (1988) studied second-grade children's concepts of ten and found that their understanding could be divided into three levels – ten as a numerical composite, ten as an abstract composite unit, and ten as an iterable unit. In order to fully understand place value, one must view ten as an iterable unit; that is, being able to view ten as both a single entity as in a group of ten units or as a group of individual objects. However, for many children, their understanding of ten is that of a numerical composite in which they focus on the individual ones that make up a group rather than seeing the group as a single entity (Baroody, 1990). Fuson (1990) argues that textbooks are to blame for children's lack of understanding ten as an iterable unit, noting that textbooks use a skills analysis approach to introduce place value with a primary focus on procedures for addition and subtraction. She suggests that textbook tasks should be reallocated in order to relate the unitary conceptual structures that children understand in the beginning to multiunit structures that they need to develop in order to extend their understanding to work with multi-digit numbers.

The question then arises as to how one should help children best develop the ability to move flexibly between different units. Digit-correspondence tasks ask students to construct meanings for the individual digits in a multi-digit number (Ross, 1999). An example of such a task would ask students to identify the meaning of the numeral 5 in the number 25 and then determine what the numeral 2 represents. Ross (2002) used these tasks to develop digit-correspondence lessons in which students worked activities that ultimately had them focus on the meanings of the digits in a two-digit number. Prior to the lessons, when the students were asked to identify what the 3 represented in a group of 35 beans, only 19 percent of the subjects provided correct explanations. Post-assessment results revealed that of the fifty-six unsuccessful attempts in the pre-test, thirty-nine, or 70 percent, of these students were now able to correctly identify the meaning of the digit after working lessons that involved digit correspondence tasks. Cotter (1996) also achieved similar results when she worked with first grade classrooms using multiple forms of manipulatives to help children break apart multidigit numbers and recognize that each digit represented a different unit.

Regrouping

Along the lines of unitizing, regrouping is another key component needed to develop a complete understanding of place value. Regrouping involves being able to express a number using multiple representations of group sizes. For example, the number 287 is typically understood to represent 2 hundreds, 8 tens, and 7 ones. However, this is not the only way to represent this number as it could also be expressed as 1 hundred, 18 tens, and 7 ones, or simply as 287 ones. Focusing on the different ways that a number can be expressed through regrouping helps children to continue to develop multiunit conceptual structures and also prepares them for performing calculations (Fuson, 1990). Resnick (1983) distinguishes between unique and multiple partitioning to describe the process of regrouping multidigit numbers to form equivalent representations. She argues that each multidigit number has a unique partitioning, and this grouping is learned in the initial stages of working with multidigit numbers. For the number 287, she would define the unique partitioning as 2 hundreds, 8 tens, and 7 ones. As children work more with multidigit numbers, Resnick states that the skills associated with multiple partitioning develop as they begin to recognize different, yet equivalent, representations for the same number.

While it is important to help children understand that multidigit numbers can be expressed in multiple ways using regrouping, this skill does not come easily for children. Bednarz and Janvier (1988) completed a three-year longitudinal study with children beginning in first grade to understand how children understand groupings related to multidigit numbers. Among other conclusions, they found that children view a number as several digits aligned in a particular order. Therefore, they are not able to correctly interpret what each digit position represents in terms of groupings. Consequently, many children cannot break apart groupings and do not see the need for regrouping in a given task.

One of the reasons why children are not able to regroup numbers successfully may be due to the fact that their teachers are also unsuccessful at this task. Thanheiser (2009) studied preservice teachers' conceptions of multidigit numbers in the context of the standard algorithms taught for addition and subtraction. Working with a small group of fifteen preservice teachers who had not attended a mathematics methods course, she found that 67% of the preservice teachers held incorrect conceptions regarding multidigit numbers. Specifically, she found that seven of the participants possessed a concatenated digits plus conception in which they viewed at least one digit of the number with an incorrect unit. For example, a preservice teacher with this conception might view the number 287 as 200 ones, 8 ones, and 7 ones. Furthermore, three of the preservice teachers held a concatenated digits only conception in which they viewed multiple digits incorrectly, as in stating the number 287 as 2 ones, 8 ones, and 7 ones. Possessing an incorrect conception of the groupings associated with a multidigit number presents further difficulty in correctly teaching children to regroup and learn to perform calculations, ultimately resulting in a procedural understanding of place value rather than a conceptual one.

Number Relationships

The base-ten number system is structured hierarchically based on powers of ten. As a result, a ten-to-one ratio between adjacent unit types exists. That is, in a given number, each new place value position created by moving to the left is ten times greater than the previous position. Past research indicates that not all students or preservice teachers recognize this underlying structure of the base-ten number system (Chick, 2003; Kamii, 1986). To study preservice teachers' understanding of place value, Chick asked two different groups of participants to explain why a zero is added to the end of a whole number when multiplying by ten. One group of participants consisted of preservice secondary mathematics teachers (DipEd cohort) while the other group was comprised of preservice primary teachers (BEd cohort). Surprisingly, 62% of the members of the DipEd cohort provided unsatisfactory responses compared with 68% of the BEd cohort. This result seems to suggest that while participants in the DipEd cohort had taken more content courses in mathematics, they did not possess a higher understanding of number relationships than participants from the BEd cohort.

Since number relationships are not easily grasped by teachers or students, activities designed to foster an understanding of the number relationships inherent within the base-ten number system can help develop a stronger understanding of place value. Thornton, Jones, and Neal (1995) suggest that a hundreds chart is a powerful way to help children begin to recognize number relationships and patterns even in the initial stages of learning to count. Van de Walle (2007) suggests using a hundreds chart to help children establish relationships with landmark numbers, or multiples of ten. Doing so reinforces the ten-to-one ratio that forms the structure of the number system and prepares students for success in operating with multidigit numbers.

Assessing Conceptual Understanding

A conceptual understanding of mathematics involves an understanding of the interrelations between pieces of knowledge in mathematics. Therefore, conceptual understanding is often associated more with intuition than with possessing the ability to perform calculations, and is therefore more internal (Montfort, Brown, & Pollock, 2009). Consequently, developing the means to assess conceptual understanding is much more difficult as one cannot just simply apply an algorithm to work out a problem and show that he or she has a conceptual understanding of mathematics.

A large body of research, particularly within the science domain, has been developed to understand ways to assess conceptual understanding. These methods include using task-oriented interviews (Montfort, Brown, & Pollock, 2009; O'Kuma, Maloney, & Hieggelke, 2003; Rittle-Johnson & Alibali, 1999) along with concept problems (Kifowit, 2004; Ross, 2009). To assess conceptual understanding among a large group of individuals conceptual diagnostic tests have been shown to be helpful (Beichner, 1994; Hestenes, Wells, & Swackhamer, 1992; Thornton & Sokoloff, 1998; Zelik, 1993). Finally, concept maps, which will be used in this study, have also been employed to understand how individuals connect ideas related to a central topic together.

Concept maps have been used within a variety of subject areas to assess students' conceptual understanding of a particular topic since they provide a way to "see" how students organize ideas together as well as recognize misconceptions the students may hold (Baroody & Bartels, 2000; Cassata, Himangshu, & Iuli, 2004; Shavelson & Ruiz-Primo, 2000; Williams, 1998). A concept map is a visual representation of the connections that a student has made surrounding a given topic. Developed by Novak and Gowin (1984), a map typically illustrates a hierarchical structuring of the prevailing concepts centered on a main topic with bi-directional links that describe the connections between the concepts. Students can usually learn to construct a concept map in a short amount of time with limited practice.

The difficulty that goes along with using concept maps to assess conceptual understanding is the decision on how to analyze them to collect the necessary data. A review of the literature related to analyzing concept maps reveals that there are two main approaches: quantitative and qualitative analysis (Shavelson, Lang, & Lewin, 1993). Quantitative approaches to analyzing concept maps typically involve the use of rubrics designed to look for specific features within the constructed map (Bartels, 1995; Cronin, Dekker, & Dunn, 1982; Novak & Gowin, 1984). Some of these features include assessing the links between concepts, the hierarchical structure of the overall map, along with the ability of the map to communicate the individual's understanding. The features are then scored based on their ability to correctly illustrate the related ideas, and the sum of the scores for all of the features is then calculated to arrive at a final grade for the map. This method of analysis does have its limitations. For instance, if the researcher is only counting the number of valid links between concepts, he or she may miss the links that are invalid. These links could provide a better picture of an individual's overall understanding, or lack thereof, regarding the topic of interest.

Nicoll, Francisco, and Nakhleh (2001) implemented a three-tier system used to analyze concept maps to assess students' understanding of chemistry concepts. This assessment system focused primarily on the links used to connect the related concepts, analyzing them at three levels, namely utility, stability, and complexity. They first assessed the utility of the links as useful, wrong, or incomplete. Next, links were coded for their stability, defined as how confident the student was in linking the two concepts together. If students were not confident in a particular link between two concepts, they were instructed to use a dashed line to form the connection rather than a typical solid line. A third level analysis assessed the links for their complexity, based on whether the link was used to illustrate an example or explain the relationships with other links included in the map. Nicoll, Francisco, and Nakhleh asserted that these links were the most helpful in assessing students' understanding as they provide an overall picture of the connections between concepts.

A qualitative approach to analyzing concept maps was developed by Kinchin and Hay (2000). Their method focused on categorizing concept maps into three main structures – spoke, chain, and net. An example of each type of map is given in Figure 2.1 on the following page. They concluded that the structure of the map was helpful in assessing a student's ability to assimilate new knowledge into their existing structure. A student that possesses a net structure of knowledge related to a particular topic has the most flexible understanding, as he or she can access different concepts through several different routes, a characteristic not seen within the spoke and chain structures.

Finally, Lapp, Nyman, and Berry (2010) used concept maps to assess students' understanding of concepts related to linear algebra. They developed two new techniques useful for analyzing concept maps. The first method involves recognizing "clumps" within a concept map. As they viewed videos of students constructing their concept maps, they noticed that the students would construct different portions of their maps at different times. They called each portion a "clump," recognizing it as a group of concepts that fit tightly together. They categorized each clump according to one of the structures developed by Kinchin and Hay (2000) and counted the number of concepts within each clump to assess the depth of understanding.

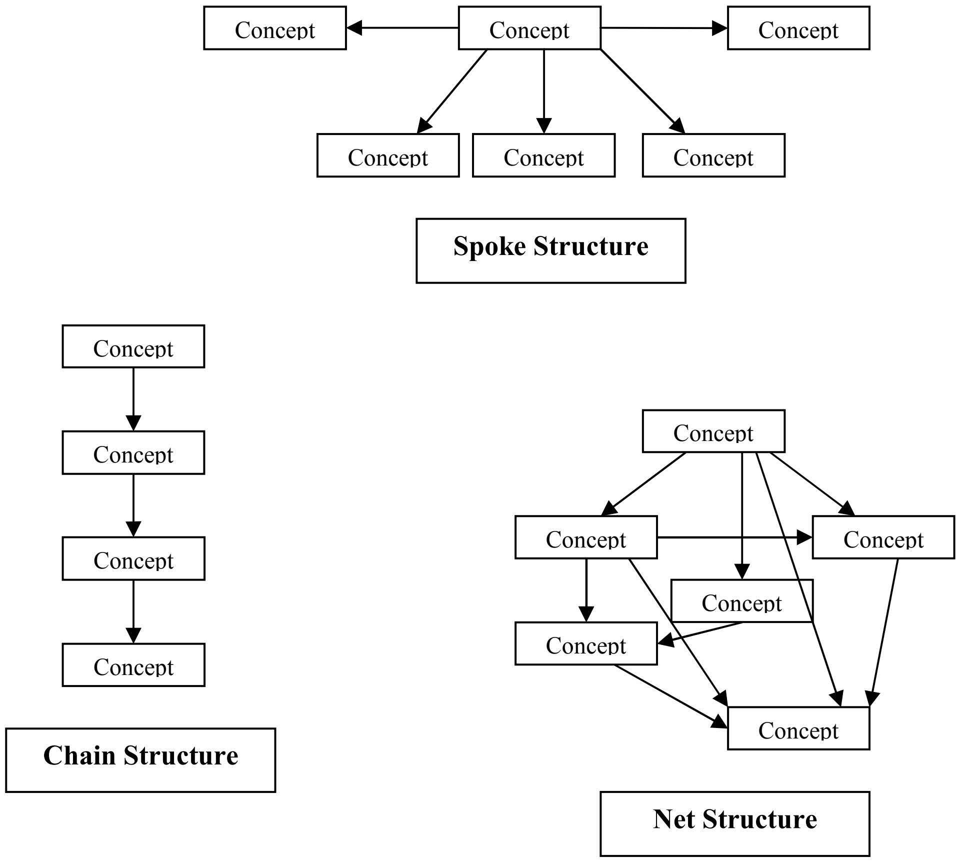


Figure 2.1. Concept Map Structures

The second technique that Lapp et. al. (2010) developed was the use of an adjacency matrix used to assess the richness of the connections between the concepts within a particular clump. They viewed each concept as a vertex of a graph and inserted a value of 1 into the matrix for the ith row and jth column location if there was an edge between vertex i and j. When no edge existed between two vertices (or concepts), a value of 0 was inserted at that location in the matrix. Squaring the adjacency matrix allowed the researchers to analyze the number of concepts directly connected to a given vertex or pair of vertices. An example illustrating these ideas is provided in Figure 3.1, found in the following chapter, as it relates to how I specifically used this technique in this study.

Theoretical Framework

Substantive Theory

Three theories, related to place value understanding, will be used to inform my approach to this study. These theories include Pirie and Kieren's (1994) model for the growth of mathematical understanding, Ozdemir and Clark's (1997) categorization of conceptual understanding from a knowledge-as-elements perspective, and Vergnaud's (2009) theory of conceptual fields. Pirie and Kieren's model will be used to highlight how place value understanding develops through eight distinct phases. Recognizing conceptual understanding from a knowledge-as-elements perspective will help explain how teachers view the concepts related to place value within their own minds. Finally, conceptual field theory will help connect the concepts that teachers hold related to place value with the situations that bring meaning to them.

Pirie and Kieren's Model for the Growth of Mathematical Understanding

As Pirie and Kieren (1994) observed children learning mathematics through their own research, they were forced to ask the question: "What *is* mathematical understanding?" They answered this question by developing a model to describe how mathematical understanding develops through eight distinct levels: (a) primitive knowing, (b) image making, (c) image having, (d) property noticing, (e) formalizing, (f) observing, (g) structuring, and (h) inventizing.

The beginning stages of understanding mathematics are found at the level of primitive knowing. Pirie and Kieren (1994) are careful to note, however, that "primitive here does not imply low level mathematics, but is rather the starting place for the growth of any particular mathematical understanding. It is what the … researcher assumes the person doing the understanding can do initially" (p. 170). The second stage, image making, involves physically creating images, whether through the use of manipulatives or paper-and-pencil drawings, to describe the new situation that the learner is trying to understand. As the learner becomes more comfortable within the new situation, he or she does not have to rely on the physical objects to make meaning. Instead, he or she creates a mental construct of the situation that can be relied upon as further understanding develops. Consequently, the learner has progressed into the third stage of mathematical understanding, known as image having. As the learner continues to work with the created images, he or she begins to recognize properties inherent to all of the images, moving into the stage of mathematical understanding labeled as property noticing. For example, if a child has created mental images of what the fractions 1/2, 2/4, and 3/6 represent, he or she would move into the stage of property noticing as he or she recognizes that all of these fractions are equivalent. At this point, the learner is moving from a concrete understanding of mathematics to one that is more abstract (Pirie & Kieren, 1994).

The fifth level of understanding, known as formalizing, occurs when the learner is able to recognize that the properties inherent to the images he or she created apply to all situations within a similar context. It is only at this point when the learner is "capable of enunciating and appreciating a formal mathematical definition or algorithm" (Pirie & Kieren, 1994, p. 171). The act of observing, the sixth stage of mathematical understanding, happens when the learner is able to reflect on what he or she has learned within the situation and recognize patterns inherent to the topic that he or she is attempting to understand. The final two stages of mathematical understanding, structuring and inventizing, occur as the learner is able to view his or her formal observations as a theory and then extend this understanding to new situations (Pirie & Kieren, 1994).

While they have situated the growth of mathematical understanding into eight distinct phases, Pirie and Kieren (1994) stress that they do not view the development of mathematical understanding as a "monodirectional process" but rather "as back and forth movement between levels and it is thus that we characterize understanding as a dynamic and organizing process" (p. 172). They describe one feature of their theory, folding back, as the process a learner must go through when faced with a problem that cannot be solved immediately. At this time, the learner must fold back to a previous level of understanding in order to begin figuring out how to solve the problem. While the learner remains at the inner level of understanding, he or she is strengthening his or her knowledge at this level and therefore exits the folded back level with a deeper understanding, motivated by the new problem.

Pirie and Kieren's (1994) model used to describe the growth of mathematical understanding can be directly applied to the concepts related to place value. Children enter elementary school with an informal sense of number and quantity (Briars & Siegler, 1984;

Carpenter, et. al., 1999; Schaefer, et. al., 1974); thus, children bring with them a primitive knowledge of place value. As children start to work with manipulatives, they begin to create images for viewing ten individual objects as one group of ten (image making) and eventually are able to see a two-digit number written down and recognize the number as consisting of groups of ten along with singles (Cobb & Wheatley, 1988; Van de Walle, 2007). Therefore, their understanding has progressed to the stage of image having. Continued work with multi-digit numbers provides children an opportunity to recognize common properties among them (property noticing) and progress into the later stages of the model.

However, just as Pirie and Kieren (1994) suggest, place value understanding is not "monodirectional" as new situations force children to fold back to a previous level of understanding. For example, as children start to understand how to add multi-digit numbers, they may need to fold back to the level of image making to represent each multi-digit number using concrete objects prior to adding them together. Similarly, as the number system is extended to include decimals in the later elementary grades, children may be forced to fold back to their primitive knowledge of place value related to whole numbers in order to understand how to apply their knowledge to this new group of numbers.

As teachers work within a new number system, they, too, will have to rely on their primitive knowledge of place value within the base-ten number system. Assuming Pirie and Kieren's (1994) model accurately depicts the growth of mathematical understanding, applying the model within the context of place value will provide insights into how teachers' place value understanding develops. This does not suggest that learning the Orpda number system will be an easy process, but if a teacher possesses a deeper, more robust knowledge of place value prior to being introduced to Orpda, he or she will be able to rely on this knowledge and make connections between the base-ten and Orpda number systems.

Conceptual Understanding from a Knowledge-as-Elements Perspective

Margolis and Laurence (1999) suggest that there is no universal definition for conceptual understanding since it is an internal, intuitive process that happens in the minds of individuals. Nonetheless, if something is understood conceptually, it has become part of the learner's knowledge and is believed to be true; it is not just simply a fact to be remembered (Montfort, Brown, & Pollock, 2009). Ozdemir and Clark (2007) characterize the two competing theoretical orientations related to conceptual understanding as knowledge-as-theory perspectives and knowledge-as-elements perspectives.

Researchers that hold a knowledge-as-theory perspective argue that learners hold naïve theories based on their experiences, and they use these theories to extend their knowledge across different domains. They argue that conceptual understanding occurs as learners recognize a conflict in their current theories and consequently must develop new mental models to accommodate for the new information (Chi & Roscoe, 2002; Posner, Strike, Hewson, & Gertzog, 1982; Vosniadu, 1994).

On the other hand, researchers of the knowledge-as-elements perspective maintain that students' knowledge base consists of multiple, independent elements (Clark, 2006; diSessa, 2002). diSessa defines these elements as phenomenological primitives, or p-prims and believes that these p-prims are loosely connected into a larger knowledge structure. The process of conceptual understanding involves strengthening the knowledge structure by refining, reorganizing, and revising the elements and their connections with other elements. While both perspectives agree that conceptual understanding occurs gradually over time, researchers that hold a knowledge-as-elements perspective view it as “a piecemeal evolutionary process rather than a broad theory-replacement process” (Ozdemir & Clark, 2007, p. 355).

Past research related to teachers’ understanding of mathematical concepts reveals that their understanding is fragmented with a focus on procedures rather than concepts (Chick, 2003; Ma, 1999; Thanheiser, 2009). Consequently, as teachers approach this study, they will bring with them some prior knowledge related to place value. I believe that this primitive knowledge, as defined by Pirie and Kieren (1994), will consist of multiple, independent elements related to place value, such as understanding how to count or how to write multi-digit numbers. As a result, this research study will view conceptual understanding from a knowledge-as-elements perspective and will seek to recognize how teachers organize their knowledge of place value before the study begins and again at the conclusion of the study.

Vergnaud’s Theory of Conceptual Fields

Vergnaud (2009) defines a conceptual field as:

> A set of situations and concepts tied together. By this, I mean that a concept’s meaning does not come from one situation only but from a variety of situations and that, reciprocally, a situation cannot be analyzed with one concept alone, but rather with several concepts, forming systems. (p. 86)

In other words, Vergnaud asserts that it is the learning situations that provide meaning to the related concepts, and collectively the concepts and situations combine to form a learner’s conceptual field within a particular knowledge domain. This study will use concept maps to begin to understand how teachers view the ideas related to place value. One of the purposes for using concept maps to assess the teachers’ conceptual understanding of place value is that the maps will allow me to recognize the relationship between the concepts and situations that bring them meaning and will therefore provide a visual representation of each teacher's conceptual field within the mathematical domain of place value.

According to Vergnuad, conceptual fields are made up of what he refers to as theorems-in-action and concepts-in-action. Within the domain of mathematics, a theorem is a sentence or proposition that is proven to be true. Theorems-in-action, on the other hand, are believed to be true by the learner until a counterexample can be found. Therefore, theorems-in-action can be true or false. Concepts-in-action are neither true nor false, but rather are referred to as relevant or irrelevant to the theorem. Consequently, Vergnaud argues that theorems cannot exist without concepts and concepts cannot exist without theorems; it is through the situations in which they are used that theorems and concepts have meaning (Vergnaud, 2009). Within a concept map, the major ideas related to the topic, or theorems-in-action, are included along with their related concepts-in-action. All of the ideas are then connected together with labels on each of the links that allow the learner to illustrate how he or she understands the relationships. Consequently, Vergnaud's theory of conceptual fields will not only help me to realize the conceptual fields that teachers have developed about place value, but will also provide one way to understand how they organize all of the ideas together based on their experiences in the study.

Summary

Three theories will be used to inform the study, namely Pirie and Kieren's model for the growth of mathematical understanding (1994), Ozdemir and Clark's categorization of conceptual understanding from a knowledge-as-elements perspective (2007), and Vergnuad's conceptual field theory (2009). As the teachers work through activities focused on the four key components of understanding place value – counting, unitizing, regrouping, and number relationships—as defined by Jones et. al (1996), their unfamiliarity with the Orpda number system will force them to return to their primitive knowledge of place value within the base-ten number system. Consequently, their understanding of place value will progress through many of the stages described by Pirie and Kieren (1994). During this time, the teachers' primitive knowledge of place value may be challenged, and viewing that change from a knowledge-as-elements perspective will provide me with an opportunity to understand those changes and how the teachers organize the new knowledge into their thinking about place value. Finally, Vergnaud's conceptual field theory will help me understand the relationship between the experiences the teachers have with Orpda and their understanding of place value, as illustrated through their concept maps. Furthermore, a review of the literature related to the key components of understanding place value and the modes for assessing conceptual understanding will also inform the design of the study and the methods of data that are collected.

Chapter III

Methodology

The decisions regarding the methodology of a research study are critical and can influence the results gleaned from the study. This chapter will first describe the theory used to inform the methodological decisions made for this study. Next, a description of the research site and sample that was used throughout this study along with the Orpda activities that the participants took part in during the mathematics education course will be presented. The procedures for data collection will be defined and the modes for analyzing them will also be explained. The chapter will conclude with a discussion of the elements included in the study to establish trustworthiness and credibility, along with a focus on the ethical considerations for the research.

Methodological Theory

Guba and Lincoln (1994) describe a paradigm as a worldview describing the nature of the world, a person's place in it, along with their relationship to the world. I approached this research study from a constructivist paradigm. I believe that individuals construct their own realities of the world around them, and these realities are shaped by individuals' experiences and can be changed with the acquisition of new information (Guba & Lincoln, 1994). Within a constructivist paradigm, the researcher aims to understand the meanings individuals construct rather than trying to understand the world itself (Rubin & Rubin, 2005).

I envisioned my role as an observer attempting to understand the knowledge teachers have and the meanings they have constructed regarding place value. My goal was to develop a deeper understanding of the teachers that participated in this study and how their place value understanding developed. From a constructivist perspective, knowledge is gained through the use of qualitative methods since this type of data allows the researcher to maintain the complexities of the environment in which the data was collected (Creswell, 2007). Furthermore, qualitative inquiry provides the researcher with an opportunity to fully understand how the participants of the study make sense of the world around them (Hatch, 2002). Since this study is focused on conceptual understanding which is unique to each individual, employing data collection techniques that are qualitative in nature allowed me to “see” into the teachers’ minds and develop a better understanding of how their place value knowledge developed throughout the course of the study.

My research question and sub-questions guided my decision as to how I structured this study. The research question of interest in this study is: How does Orpda relate to teachers’ conceptual understanding of place value? To attempt to answer this question and the three sub-questions, I used an instrumental case study approach as defined by Stake (1994). When using an instrumental case study approach, “the case is often looked at in depth, its contexts scrutinized, its ordinary activities detailed, because this helps us pursue the external interest” (Stake, 1994, p. 237). For this study, the case was considered to be the Orpda numeration system developed by Dr. Theresa Hopkins and Dr. Jo Ann Cady (2007) as it was used within the context of a mathematics education course with the intent of understanding the relationship between Orpda and the teachers’ conceptual understanding of place value.

Using a case study approach fits with my approach to this research from a constructivist perspective. By focusing on a small number of participants, I was able to understand how the participating teachers made meaning of the concepts related to place value and analyze the relationship that Orpda had with their understandings. Furthermore, since case studies are bounded within the context of the study, as I examined the data collected from the participants, I was able to maintain the teachers' unique representation of their place value knowledge and develop a rich description of how they made meaning within this context.

When conducting a qualitative study, the researcher both guides and shapes the overall result. As a result, the beliefs that the researcher brings to the study ultimately informs the study through the researcher's decisions, including what to look for throughout the study and what to report in the end (Rubin & Rubin, 2005). Since this can have a significant impact on the study, it is important that the researcher clearly state his or her biases. Consequently, the remainder of this section will focus on the biases that I brought to this study.

Reflexivity Statement

I entered the doctoral program in mathematics education at this university with a strong background in mathematics. I received my bachelor's degree in mathematics with a focus on pure mathematics that included taking courses in real analysis, modern algebra, and number theory. Upon completion of my bachelor's degree, I obtained my master's degree in mathematics as well and completed my master's thesis in the area of graph theory. Needless to say, I enjoyed studying mathematics and the challenge that each new problem presented. However, after finishing my master's degree, I decided to take a break and teach mathematics at the community college level in order to better decide my next direction. I immediately fell in love with teaching and truly enjoyed the atmosphere of teaching at the post-secondary level. In order to make this my future career, I knew that I needed to obtain a doctoral degree. While I loved studying mathematics, I had realized that my true passion lay in teaching mathematics and I therefore decided to focus on mathematics education for my final degree.

With all of the upper level mathematics courses I had taken throughout college and beyond, the concept of place value was, in my mind, a very distant memory. I did not really think about how it related to the classes I had more recently completed. I enrolled in a mathematics education course for my doctoral degree focused on teaching elementary mathematics. While I had no idea of what to expect since I had not taught elementary mathematics, I thought it would be interesting to better understand teaching at this level. On the first day of class I was introduced to Orpda and from that point forward, I was embarrassed to admit that I had two degrees in mathematics, but had absolutely no idea what the instructor was talking about as she introduced this new number system.

After I worked through the many Orpda activities that would eventually come to be used in this study, I quickly started to learn that there was a lot more to understanding place value than I initially had thought. Therefore, my intrigue with this subject led me to begin reading research related to how children understand place value and ways to help teachers better understand place value for themselves. During this time, I began to focus on the four main attributes of place value – counting, unitizing, regrouping, and number relationships.

Consequently, as I approached this study, I wanted to better understand how teachers thought about these four aspects of place value after being placed in an unfamiliar learning environment. My goal for this study was to recognize one way of helping teachers strengthen their knowledge of place value and understand what a critical role it plays in future mathematics that their students will learn. My own experiences with Orpda helped me to better shape and inform this study, as well as understand what the teachers were expressing through their reactions.

Description of Research Site

This research study was carried out at a large, research university located in the southeastern United States. The education program at this university consists of required undergraduate coursework that preservice teachers must complete along with a two-semester graduate internship. Upon completion of all requirements, preservice teachers are licensed to teach within the state in which the university is located.

In addition to granting initial licensure to teach at the K-12 level within the state, the teacher education program also offers degrees in other areas of education as well as at the graduate and doctorate levels. Consequently, in a given semester, a mathematics education class may consist of a mix of students with varied teaching backgrounds ranging from no experience in the classroom to multiple years of teaching, possibly at different academic levels.

Description of Sample

For the purposes of this study, the term *teacher* will refer to both preservice teachers and in-service teachers, with the viewpoint that from the time an individual enrolls in a teacher education program and throughout his or her teaching career, he or she is placed on a continuum of the teaching profession, while the term *instructor* will refer to the individual presenting the Orpda ideas and activities. While there were 16 teachers enrolled in the mathematics education graduate course that was offered as an elective during the summer of 2010 at the described university, only 13 of the teachers provided informed consent for their materials to become data and were therefore the participants of this study. The sample of participants contained a mix of six elementary school teachers, one middle school teacher, four high school teachers, and two college teachers. Teaching experience for the participants ranged from those with no experience who were preparing to start a one-year internship, up to those with three years experience in the classroom. Math anxiety levels among the teachers ranged from no math anxiety to medium anxiety. Class meetings for the course were held five days a week for an hour and a half each day with the course lasting a total of five weeks. However, the Orpda number system was only presented during the first full week of classes.

Description of Orpda Activities

The instructor of the mathematics education course used for this study agreed to implement Orpda as part of the course requirements. Therefore, all teachers enrolled in the course participated in the activities associated with Orpda. For the first full week of classes, the instructor of the course presented the Orpda numeration system to the teachers. Throughout the week, the teachers worked together in groups of four, and they remained sitting with the same group throughout the entire week of activities. The presentation of Orpda resembled the way that the base-ten numeration system is presented in elementary classrooms; thus, the teachers first learned the oral and written language associated with Orpda and then worked through numerous activities designed to enhance their knowledge of the key components related to a conceptual understanding of place value – counting, unitizing, regrouping, and number relationships. These activities were originally developed by Van de Walle (2007) and Super Source ETA Cuisenaire (2000) to work in the base-ten number system and were adapted to work with Orpda. As the teachers worked through each activity, they were asked to reflect on their experiences and think about essential questions focusing on place value. Table 1.2 on the following page summarizes the schedule of activities conducted each day during the week of Orpda, along with the related key components of understanding place value, identified from the literature, that were emphasized by the activities.

Data Collection Procedures

To address each of the three sub-questions related to the research question of how Orpda relates to teachers' conceptual understanding of place value, qualitative methods were employed

Table 3.1
Schedule of Activities During the Week of Orpda

Day	Activities	Focus
Monday	Oral Counting, Introduce symbols and words, Dot Plates, Overhead Frames, Flub Frames, @~~ chart	Counting Anchors to Flub and Atty Patterns Number Relationships
Tuesday	How Many in All? How Many? Fill the Flub Frames	Counting Unitizing Patterns
Wednesday	Language Patterns Counting Bags Race for a Flat How Many Ways?	Counting Unitizing Regrouping Patterns
Thursday	Orpda Word Problems Addition Grids	Unitizing Regrouping Computation Number Relationships
Friday	Invented Algorithms for Addition and Subtraction	Regrouping Computation Number Relationships

throughout the study in order to understand the ways that the teachers construct meaning of place value concepts within this context. Data was collected from several different sources, including concept maps, discussion board questions and activity reflection guides, transcriptions from classroom discussions, an interview with the instructor, and teacher demographic sheets as a means to strengthen the study (Yin, 2009). This section will illustrate these sources and how they were used within the study.

Concept Maps

The mathematics education course that was used for this study began during the middle of the week within the month of June 2010. Therefore, in order to allow five consecutive days for the teachers to work in Orpda, the presentation of Orpda began on Monday of the first full week of classes. During the class meetings prior to the start of Orpda, the teachers were introduced to concept maps. Concept maps are visual representations of an individual's knowledge structured around a central topic (Novak & Gowin, 1984). These maps may follow many different forms as there is not one defined way in which to draw one (Mintzes, Wandersee, & Novak, 2001).

While I did not want to bias their responses, I felt that some instructional training was necessary to provide the teachers with an understanding of some of the major components of a concept map. Consequently, I conducted a twenty minute training session during class that involved showing the teachers some examples of concept maps about topics not related to place value followed by a brief discussion of the characteristics inherent among all of them. The teachers were then instructed to create their own concept map illustrating their understanding about place value for homework prior to the first class meeting of Orpda.

The teachers constructed a second concept map which was used to compare with their first map in order to recognize changes within their understanding of place value. While the teachers concluded their work with Orpda at the end of the first full week of classes, they were not asked to construct their second concept maps until the end of the course. The reasoning behind this approach is that conceptual understanding develops over time (Ozdemir & Clark, 2007). Therefore, allowing four weeks of time to pass before they constructed another map provided the teachers with an opportunity to reflect on what they had experienced during Orpda. As with the first concept map, the teachers were asked to construct their second maps for homework towards the end of the course.

Discussion Questions and Activity Reflection Guides

At the conclusion of each class meeting during which the teachers worked with Orpda, they were asked to respond to a couple of discussion questions. These questions were posted on Blackboard, an online course management system to which all of the participants both had access and were required to respond to outside of class before the next class meeting. Some of the questions to which the teachers responded were set up in Blackboard as private blogs, meaning only the instructor of the course and I could see their responses. At other times, the teachers were divided into two open discussion groups and they were asked to post their responses as well as respond, as necessary, to other teachers' comments within their group. The questions were designed to help the instructor and (or) the researcher understand how the participants were feeling as they were introduced to Orpda and how (if) their understanding about concepts related to place value changed throughout the week. A list of the discussion board questions along with how the question was set up in Blackboard can be found in appendix A.

Several benefits can be found through the use of an online discussion board to collect qualitative data for this study rather than conducting formal interviews with each teacher. First, the use of an online discussion board allowed me to collect data from all of the participants regarding their experiences with Orpda rather than just a small sample of them. Second, the participants may have felt more comfortable sharing their experiences through an online environment rather than being face-to-face with an interviewer. Third, online discussion boards allow the conversations held during a particular class meeting to be extended beyond the classroom (Dutt-Doner & Powers, 2000). Since the participants were only working with Orpda for five classes, the online discussion questions allowed additional data to be collected rather than limiting it only to what happened during class. Finally, the online discussion questions help to foster a reflective discourse to enhance the data (Mason, 2000; Nicholson & Bond, 2003). Previous use of Orpda with teachers at conferences and other professional development settings has revealed that working with a new number system presents many initial challenges, making the time for reflection a crucial and necessary component to this study.

In addition to responding to discussion board questions throughout the week, the teachers were also asked to complete activity reflection guides in order to consider their thoughts and experiences related to each Orpda activity. A copy of the activity reflection guide can be found in Figure 4.4 on page 64.

Classroom Observations

One of the key sources of evidence when doing a case study involves observing the natural setting of the "case" (Yin, 2009). For this study, Orpda represents the case being studied. Consequently, observations of the teachers working in Orpda need to be included as a means for collecting data. During each of the five class meetings when the teachers worked in Orpda, I assumed the role of an observer. My purpose during this time was to take detailed field notes of the whole class discussions along with the group discussions that occurred while the teachers worked in Orpda. I also used a digital audio recorder to capture the conversations that occurred, and the recording transcriptions were used to supplement my field notes in order to prevent me from missing any key moments throughout the class. The field notes and audio recording transcriptions were used to supplement the data collected from the discussion questions and activity reflection guides in order to better understand the progression of the teachers' place value understanding throughout their time with Orpda.

Instructor Interview

Since I was only present for the first week of classes when the teachers participated in Orpda, a follow up interview with the instructor was conducted after the class had concluded and grades had been submitted. The purpose of the interview was to gain insights into how the class developed as a whole over the remaining four weeks of class meetings. In addition, the interview allowed me to gain further insights into some of the conclusions I was starting to notice through the beginning stages of the data analysis. Overall, the instructor interview served as an opportunity for me to gather any additional data that may have been useful in the later analysis that I was not present to collect myself. A copy of the questions used for the instructor interview can be found in appendix B.

Teacher Demographic Sheets

A final source of data used for this study involved information collected from the teachers enrolled in the class during the first class meeting. These information sheets collected demographic data related to each teacher, including information about the high school and college mathematics courses they had taken, along with their reported level of math anxiety. In addition, the teachers were asked to provide information regarding their desired grade level for teaching. Finally, the information sheets asked the teachers to respond to open-ended questions regarding what mathematics means to them, along with how they view the student's and teachers' role in a mathematics classroom. A copy of the teacher demographic sheets can be found in appendix C.

The purpose for collecting the teacher demographic sheets was to provide a better understanding of the group of participants from the study. Knowing the mathematics content background for each teacher entering the study provided insights into their responses and reactions to Orpda. Furthermore, I was able to group the teachers together according to their desired grade level of teaching in order to recognize patterns within the data collected from other sources.

Data Analysis

Concept Maps

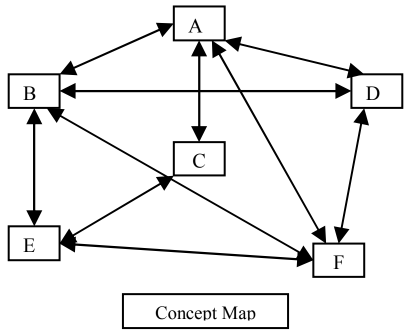
Past studies that have used concept maps to assess students' conceptual understanding within a particular domain have analyzed the maps in comparison to "expert" maps, defined as maps created by individuals that would be considered experts within the knowledge area, such as university professors (Cassata, Himangshu, & Iuli, 2004; Ruiz-Primo & Shavelson, 1996). The purpose of this study, however, was to understand teachers' conceptual fields, as defined by Vergnaud (2009) in chapter two, in the area of place value. Specifically, this study aimed to understand the relationship between Orpda and the teachers' understanding. Furthermore, in maintaining a constructivist approach to this research, I recognized that each teacher's concept map would be unique as it reflected his or her own understanding of place value. My purpose for analyzing the concept maps was to understand how the teachers connected the different concepts related to place value together as well as identify points during their time with Orpda that either enhanced their understanding or presented struggles.

Keeping the goals of the study in mind, I analyzed the teachers' concept maps constructed before and after Orpda across three different phases. For the first phase of analysis, I categorized the teachers' maps according to the categories set forth by Kinchin and Hay (2000); that is, I labeled each map as a spoke, chain, or net concept map. Illustrations of each type of map can be found in Figure 2.1 in chapter 2. Two main reasons justified my approach to categorizing the teachers' maps in this way. First, I considered the groups of pre- and post-Orpda concept maps separately and looked for common themes among the two groups. In doing so, I was able to understand how the teachers as a group connected the ideas associated with place value together in their minds both before and after Orpda. This analysis also allowed me to recognize any differences I saw among the two groups which would indicate changes in the way the teachers thought about place value after their experiences with Orpda. Second, I compared each teacher's individual pre- and post-Orpda concept maps in order to recognize any individual differences in the way they connected the related place value ideas together.

The second phase of analysis of the pre- and post-Orpda concept maps involved recognizing key concepts that the teachers identified in their maps as being important to understanding place value. Again, I looked at the pre- and post-Orpda concept maps separately in order to make comparisons among the two groups and recognize any changes in how the teachers viewed the essential elements related to place value. This phase of analysis allowed me to better understand what aspects related to place value the teachers felt were most important, and consequently, what aspects they might emphasize in their own teaching of place value. In addition, I was able to recognize whether their understanding about place value was of a more procedural or conceptual nature both before and after Orpda.

Finally, I analyzed the pre- and post-Orpda concept maps using a technique developed by Lapp, Nyman, and Berry (2010). This study defines conceptual understanding as knowledge that is rich in connections with many pathways between ideas and concepts. Consequently, this final phase focused solely on the connections that the teachers made between the concepts they viewed as essential to understanding place value. To assess the richness, or strength of the connections between the concepts within the map, an adjacency matrix was constructed from each teacher's pre- and post-Orpda concept map. Adjacency matrices are commonly used in the area of graph theory to analyze the connections made between the vertices within a given graph (Chartrand, 1984). In this context, each concept was considered as a vertex of a graph and any two concepts that were linked together in the map shared an edge between them. If two concepts were connected, then a value of 1 was entered into the matrix in the *i*th row and *j*th column. Otherwise, if no link existed, a value of 0 was recorded. Once the adjacency matrix for a teacher's map was created, I then squared the matrix. I considered two main features of the resulting squared adjacency matrix – the value of the entries along the main diagonal and the value of the entries below the main diagonal. The entries on the main diagonal of the squared adjacency matrix represented the number of concepts that a given concept was directly connected to in the map. In addition, the entries of the matrix below the main diagonal represented how many indirect connections existed between two different concepts in the map.

An illustration of an adjacency matrix as well as the squared adjacency matrix can be found in Figure 3.1 on the following page. The vertices of the map, labeled A through F, are used on the columns and the rows of both matrices. Examining the squared adjacency matrix,



Adjacency Matrix

	A	B	C	D	E	F
A	0	1	1	1	0	1
B	1	0	0	1	1	1
C	1	0	0	0	1	0
D	1	1	0	0	0	1
E	0	1	1	0	0	1
F	1	1	0	1	1	0

Squared Adjacency Matrix

	A	B	C	D	E	F
A	4	2	0	2	3	2
B	2	4	2	2	1	3
C	0	2	2	1	0	2
D	2	2	1	3	2	2
E	3	1	0	2	3	1
F	2	3	2	2	1	4

Figure 3.1. Concept Map and Matrices

the highlighted numbers down the main diagonal represent how many different concepts are connected to any given concept. For example, the number 4 found in the cell located at the intersection of row A and column A represents the four different concepts that concept A is connected to in the corresponding map. Furthermore, the numbers above and below the main diagonal represent how many indirect connections exist between any two distinct vertices. For example, the number 3 found in the cell located at the intersection of row A and column E means that there are three different ways to reach vertex E starting at vertex A, namely connecting through vertices B, C, or F. Higher numbers above and below the main diagonal indicate more connections that exist between the concepts within the map as a whole.

Discussion Questions, Classroom Observations, and Activity Reflection Guides

One of the main purposes of including discussion questions, classroom observations, and teachers' reflections within the data collection procedures was to document how the teachers' understanding of place value was changing and developing throughout their work with Orpda, rather than just before and after. To analyze this set of data, I used a coding process outlined by Strauss and Corbin (1998). A set of a priori codes related to the key components of understanding place value as outlined in the theoretical framework and related literature was used to begin the coding process of the teachers' responses. A copy of these codes is provided in appendix D I then looked for additional codes and themes present within the data and incorporated these new codes into the previous coding scheme. Upon completion of these two phases of coding, categories were developed from the codes (listed in appendix E) to gain a more complete understanding of the relationship between Orpda and the teachers' conceptual understanding of place value.

Trustworthiness and Credibility

Several methods were used to ensure that the study maintained trustworthiness and credibility. First, this research study utilized multiple sources of evidence to answer the research question (Yin, 2009), including concept maps constructed before and after the teachers worked with Orpda, along with online discussion board questions, classroom observations, reflection sheets, and interviews to document the relationships between Orpda and the teachers' place value understanding throughout the study. One of the main aspects of a case study is its focus on developing a rich, thick description of the case in its natural setting (Stake, 1994). I provided a rich, thick description of the data from the classroom observations that I attended. This description was supplemented with quotes from the participating teachers to further provide the reader with the emotional experience of the study.

In addition to providing a rich, thick description of each of the teachers' class meetings with Orpda, I also utilized member checking and external audits (Lincoln & Guba, 1990) to further provide ways of establishing credibility to the study. Once the data analysis was completed and chapter 4 was composed, I emailed the complete version of chapter 4 to each of the participants who gave me informed consent and for whom I had contact information. Furthermore, the dissertation committee served as an external audit and allowed me to gain valuable insights to help improve my research study throughout the entire process.

Ethical Considerations

Focusing on the ethical concerns continued to help me develop a valid and credible case study. There were a few elements that I included in my research study to help meet this demand. First, I obtained approval from the Institutional Review Board (IRB) at the selected university. Since the instructor of the mathematics education course being used for this study required all of the enrolled teachers to participate in the Orpda activities along with constructing concept maps and answering online discussion questions, it was important to make sure that the teachers did not feel coerced into participating. Near the end of the course, the teachers were given an opportunity to allow their information to be part of the study. At this time, the participants were informed of the purpose of the study as well as any risks that they assumed for taking part in the research. An outside person explained the study and distributed a consent form; she told the teachers that she would not tell either the instructor or me who had consented to participate until after grades had been submitted for the class. Using someone who was not the researcher or instructor, and holding the consent forms until after submission of grades, allowed the teachers to feel comfortable asking any questions about the study as well as not feel obligated to participate.

I also considered ways to protect my participants and uphold ethical concerns during the research. Throughout the entire process, I maintained confidentiality of the teachers' views and words provided through the discussion questions and reflections. This case study report also includes pseudonyms (chosen by the participants) to maintain confidentiality and I made every attempt to write up the report in a way that does not reveal their identity to the other participants. I was also open and honest with my participants throughout the study and provided them with copies of the final report, upon their request, so that they could review it and express any concerns prior to submitting the document. None of the participants had expressed any concerns regarding the document by the time the work was submitted. Finally, I made sure to follow all guidelines set forth by the university for conducting ethical research.

Summary

In this chapter, I outlined the design for investigating how Orpda relates to teachers' conceptual understanding of place value using an instrumental case study design. Conducting a case study allowed me to gain an in-depth look at how teachers thought about place value and how their thinking was challenged while working with Orpda. Concept maps were used as one of the methods for collecting data in order to understand the conceptual fields that the teachers held related to place value and how they organized their thinking about these concepts. Supplementing this data with online discussion board questions, classroom observations, reflections from the teachers, and an interview with the instructor allowed me to illustrate any relationships that Orpda had on their thinking throughout the process of the study. The results from this study can be used to further areas in the mathematics education literature related to deepening the content knowledge of teachers, as well as encouraging the use of reform based teaching in the classroom.

Chapter IV

Data Presentation and Analysis

The phenomenon under study in this project is teachers' experiences with Orpda. In this chapter, I will first establish the context in which teachers experienced Orpda--a mathematics education class. This will be followed by a thick description of teachers' experiences with Orpda, with discussion of research question 1, " What do the teachers' reflections on the instructional tasks done in Orpda reveal about their understanding of place value?," embedded. I will then discuss research questions 2, "What do comparisons of pre- and post-Orpda concept maps reveal about teachers' understanding of place value after experiencing Orpda?," and 3, "What connections do teachers make between Orpda and the Arabic number system?."

Context: The Mathematics Education Class

Prior to the first class meeting of Orpda, I met with the instructor to discuss the major ideas of place value that we wanted the teachers to think about as they worked through the different activities throughout the week. In addition, we discussed the order in which the instructor would have the teachers complete the instructional tasks in order to emphasize these key points as well as challenge their thinking about place value as the week progressed. Influenced by the major aspects of place value discussed in the research, as well as our own past experiences using the Orpda activities, we came up with four central themes important to a conceptual understanding of place value. These themes included a focus on counting, unitizing, regrouping, and number relationships inherent within the base-ten number system.

Each of the class meetings related to Orpda was taught using an instructional approach focused on questions and discussions in order to encourage the teachers to reflect on what they were learning through their experiences with the Orpda number system. For example, the instructor would ask prompting and probing questions rather than suggesting a fixed method for solving a problem. Common questions the instructor asked included, “Why do you think that?,” “Why does that make sense?,” and “Do you see any relationships between Orpda and the base-ten number system?” This constructivist approach to teaching put the emphasis of each class on the teachers’ reasoning about place value as opposed to the accuracy of their comments and answers. Some of the teachers were not familiar with this type of instruction, and, consequently, were less inclined to share their thoughts in the beginning of the week. As they grew more comfortable with their environment and understood that the instructor was using their comments to assess their understanding and inform further instruction, they relaxed and began to appreciate this method of teaching.

Upon completion of each class meeting, I met again with the instructor to discuss the events of the class. We compared notes made regarding the responses of the teachers to various questions and discussed where we thought the teachers were struggling with understanding place value. We then used this data to inform the instruction for the next class meeting. Consequently, the week of Orpda was an intense time for the teachers to reflect on their own understandings of place value and how those understandings were being challenged by working with Orpda. In addition, the week presented an opportunity for the instructor and me to focus on each teacher’s understanding of place value as it developed through each class meeting as well as gain a sense of what each instructional task revealed about this understanding.

The five-week summer session course began on a Thursday. During the first two days of class, the instructor and teachers spent time introducing themselves to one another in order to establish a relaxed classroom atmosphere in which everyone was comfortable sharing their thoughts with one another. In order to start to set the social norms for the class, the instructor recounted this experience:

"One of the teachers I had in a previous class said that in order to get the most out of my class, she had to leave her math at the door and just bring her brain. In other words, she could not let her previous math knowledge interfere with a new way of thinking that she was being exposed to throughout the class. That is exactly what I am asking each one of you to do during this class – think." After hearing this story, many of the teachers began to realize that this class might just be a little different from other classes they had taken, especially other math classes. The instructor continued to discuss that she would like for the teachers to work together in their groups to solve problems and discuss their ideas with one another as well as the remainder of the class. From this point on, a community of learners was set up in this class and each teacher worked together with the others to maintain this atmosphere.

On Friday, the second day of class, I visited and introduced the teachers to concept maps. Since I did not want to influence the decisions the teachers would make when they created their own concept maps, I showed the teachers examples of several different styles of concept maps that were created around non-mathematical subjects. After I showed the teachers the examples, I asked the teachers to discuss the similarities among all of the maps. My purpose for having this discussion was to ensure that the teachers knew what characteristics needed to be included in any concept map. The teachers noted that all of the maps contained nodes related to the topic of the map. In addition, they recognized that the various nodes were linked together using connecting lines and arrows, although each map displayed these links using various formats. Some of the maps contained many links among all of the nodes, while other maps displayed links coming only from several primary nodes that were connected to the main topic of the map. After discussing the different examples of concept maps, I told the teachers that their assignment for Monday was to create their own concept map that illustrated how they thought about concepts related to place value.

The Week of Orpda

Monday: Counting, Symbols and Representing Quantities

On a warm, summer day in early June, I made my way into the classroom located on the fourth floor of the education building on campus. I arrived early to help the instructor set out the materials that would be used for the different activities the teachers would do throughout the class meeting. In order to encourage discussion amongst the teachers, we arranged the tables to accommodate groups of teachers throughout the room, totaling four groups of four teachers at each table. In the middle of each table, we placed various manipulatives, as well as individual white boards and dry-erase markers for each teacher to use.

As the teachers arrived to class that Monday morning, they were allowed to choose where they wanted to sit throughout the room. In the back right corner of the room, a group of elementary teachers sat together, catching up from the weekend. Across from the elementary teachers in the back left corner of the room, high school teachers congregated together sharing stories of their teaching experiences. A group of four teachers with experience teaching at various levels ranging from middle school to college made up the group in the middle of the room, while the remaining four elementary teachers sat together at the tables closest to the door.

The focus of this Monday morning class was to first introduce the teachers to the Orpda number system, emphasizing the established language and symbolic notation used to express different quantities. To get the teachers thinking, as she commonly did, the instructor posed an initial question: “Would you rather have star skoobrat caret or caret skoobrat chocolate bars? And you have to tell me why.”

The teachers laughed, as they tried to make meaning of the nonsense they heard from the instructor’s question. What on earth was she talking about? As was common for this instructor, she did not provide any additional explanation to the initial question. She wanted the teachers to think and reason for themselves. After a few minutes, Scarlet, an elementary teacher who had openly participated in previous class discussions up to this point, said,

“Well, that’s like the carat in a diamond ring so…caret skoobrat chocolate bars.”

“Ok, so Scarlet is making connections to the real world by relating the caret in caret skoobrat to the carat in a diamond ring. Any others?” the instructor prompted.

“Caret skoobrat chocolate bars because c comes before s in the alphabet,” another teacher noted.

“I like chocolate,” Joe, a high school teacher from the back of the room added.

“You like chocolate, ok. So, which one would you want?” the instructor asked.

“I would want the caret skoobrat chocolate bars,” Joe answered rather emphatically.

“Ok, but the star skoobrat caret is another quantity of chocolate bars that you could have. So you have two quantities to choose from here - star skoobrat caret chocolate bars or caret skoobrat chocolate bars – and if you like chocolate then you obviously want to choose the larger quantity,” the instructor added to further explain her initial question.

“Oh, ok. So that’s a quantity, too. Then, I would want the star skoobrat caret chocolate bars because there are 3 words to describe it. It sounds like that is more,” Joe answered.

“Ahhhh, because you have 3 words to describe that one? Ok. So you think it’s going to be more because it has 3 words to describe it. Ok,” the instructor said.

The instructor allowed the teachers to continue providing reasons to support their arguments until she felt that the discussion was complete. At this point, she then began to set the stage for Orpda.

"Now, this would be similar to you talking to a kindergartner or first grader and saying, 'Would you rather have 4,576 candy bars or 10,000,503?' They are going to look at you like, what are you talking about? That does not make any sense to me. So, my reason for doing this is to increase your empathy for elementary students' struggles with learning place value. The things that we are telling them make no sense to them because they don't have these past experiences that we have with this. I would also like to deepen your understanding of place value, and, maybe not deepen your understanding, but make it more explicit, so that you say, 'Ohhhh, that's why we always do that. I didn't think about that before.' And, I want to deepen your understanding about effective place value instruction. What will help kids to learn place value?"

Counting

In a typical elementary classroom, a teacher introduces students to the language associated with the base-ten number system through rote counting. Usually this involves counting physical objects, attaching a word name to each one, with the final word name describing the quantity of objects in the collection. For example, a teacher might have a collection of three cars and hold up the first car to the class and say "one." The class would then repeat the word "one" together. The teacher would then proceed to hold up the next car and say "two" and the class would repeat the word "two." The teacher would then hold up the final car and say "three" and the class would repeat the word "three." To help the students remember the word names in order, the teacher then might ask the students to count all three cars together, in which the students would say, "one, two, three." This process would continue as the students learned additional word names associated with the base-ten number system and the appropriate sequence in which to say them.

The first day of Orpda involved introducing the teachers to the word names associated with the Orpda number system. The instructor began by counting each of the teachers in the class, associating each teacher with a different word name, and the teachers repeated each word name aloud. After several new word names had been introduced, the instructor had the teachers repeat the number sequence in order, starting from the beginning. This rote counting activity continued until everyone in the classroom had been counted. Table 4.1 below gives the word names that the teachers learned up to this point.

Once the teachers had counted up to atty, the instructor then told the teachers that the next word name in the Orpda number system was "atty-star." She then asked the teachers if they had an idea of the next word name in the sequence. One of the teachers responded correctly with "atty-at." The instructor prompted the teachers for the next two word names to which they responded with "atty-pound" and "atty-caret." At this point, the instructor had the teachers repeat the number sequence from the beginning, starting with "star" and continuing through "atty-caret."

To encourage the teachers to begin to recognize patterns within the word names of the Orpda number system, the instructor told the teachers that the next word name in the sequence was "poundy." The teachers, as a group, quickly recognized that the next word name would then

Table 4.1
Orpda Word Names Up to Atty

Star	At	Pound	Caret	Flub
Doozle	Sholt	Pouflube	Carflube	Atty

be “poundy-star” followed by “poundy-at.” Since the instructor had a sense that the teachers understood the inherent patterns between the word names in the Orpda number system, she posed the question.

> Instructor: You have already figured out some language patterns. Are there any language patterns in the Arabic numerals that we use?
> Hank: Ten, twenty, thirty, forty, fifty, sixty.
> Instructor: And how do all of the word names end?
> Hank: In –ty.
> Instructor: Any other patterns?
> Tom: The teens – thirteen, fourteen, fifteen, sixteen.
> Instructor: Alright, so we have a ten, and then ten can be a teen in thirteen and ten can be a –ty in twenty.

The rote counting activity that the teachers participated in was used to introduce them to the language associated with the Orpda number system. Upon completion of this activity, the teachers were beginning to get a sense of the importance of recognizing patterns in the language in order to help them remember the number sequence. In addition, the teachers understood that similar patterns existed in the base-ten number system, and it is important for children to recognize these patterns as they begin to learn the word names in order to count successfully in the base-ten number system.

Symbols and Representing Quantities

After the teachers participated in several minutes of oral counting, the instructor then proceeded to provide the teachers with the symbols associated with each of the word names they had learned. The instructor began introducing the symbols starting with the first word name that the teachers had learned through rote counting, star. The instructor showed the teachers a picture of a car, and underneath the picture she had the word name, star, and its symbol, *. The instructor then used the same technique to introduce the teachers to the next three symbols in the Orpda number system namely @ (pronounced “at”), # (pronounced “pound”), and ^ (pronounced "caret"). Table 4.2 below provides these word names and symbols, along with the next one that occurs in the sequence, flub.

At this point, the instructor told the teachers that there was only one symbol left in the Orpda number system and that symbol was tilde, denoted as ~. The next word name in the Orpda number system was flub, represented as *~, and the following gives the discussion that followed from the teachers regarding how to express this quantity using only the five symbols within the Orpda number system.

Instructor: Let's get a list of possible ways to represent that quantity [referring to flub] and we will talk about the reasonableness of each answer. Anybody want to take a guess?
Scarlet: *~
Instructor: [Writes symbol on the board.] Ok, anybody have another idea?
Liz: *^
Instructor: [Writes symbol on the board.] Another idea?
Kate: ^*
Tullula: @#
Hank: How about just tilde?
Instructor: Because that's the only symbol left right? Makes sense to use that. Now, the person who suggested these does not have to answer. So anybody can answer. What about this one? [pointing to the suggestion of *~]

(No answers from the teachers)

Instructor: Why didn't they just use ~? Why did they think they needed to use *~?

(No answers from the teachers)

Instructor: Hmmm….stumped you! Ok, let's move on. Why this one? [pointing to the suggestion of *^]
Hank: That's like 1 and 4.
Instructor: Ok, so these two added together represent that quantity? What about this one? [pointing to the suggestion of ^*]
Joe: Same thing

Table 4.2
Word Names and Symbols for the First Five Numbers

star	at	pound	caret	flub
*	@	#	^	*~

Instructor: Ok, so now we would have two ways to represent flub. What about this one? [pointing to the suggestion of @#]
Several teachers: That's like 2 + 3
Instructor: Ok, so same thing. @ cars plus # cars would give us that many [flub] cars. Or, we could do just this one [referring to ~] since we only have one symbol left. But this one [referring to *~] really intrigues me.
Lucy: Is that like when you are thinking in the base system that would be, if you were in base 4, 1 represents 5, that's like the base. Those are place values…so that's [referring to the ~] like…zero.
Instructor: And why does it make sense for this [referring to ~] to be like a zero?
Lucy: Because that would be like the place…if that's the 1's place and the 5's place, rather than a 1's and a 10's place.
Instructor: So, you are relating it to, in the Arabic number system in base 10, where this [referring to the ~ in *~] is the units place and this [referring to the * in *~] is the tens place.
Lucy: Yes.

From this excerpt of the class discussion, we gain insights into several aspects of how the teachers are thinking about representing the next quantity, flub, in the Orpda number sequence. Interestingly, Scarlet was the first teacher to guess and she guessed correctly. However, later on in the discussion when the instructor had the teachers discuss the reasonableness of each guess, initially none of the teachers, including Scarlet, could provide a reason for why it made sense to represent flub as *~. The next three guesses for the symbolic representation of flub seem to reveal an initial procedural understanding of place value, as the teachers gravitated towards computational reasoning for their guesses, despite the fact that they were not told to consider operations between the symbols, such as addition. Lucy's comments, however, suggest a more conceptual understanding of place value as she is able to relate her understanding of the tens and units places in the Arabic number system in order to understand Orpda. In addition, Lucy recognizes that there is a difference in the base for the Orpda number system as compared to the Arabic number system. Rather than telling the teachers that the correct symbolic representation for flub is *~, the instructor continues the discussion by then asking the teachers to consider how they would represent the next number in the sequence, namely doozle which has the symbol **. The following is a portion of the resulting discussion.

Instructor:	So, let’s not make a decision yet, but let’s go on to this one. How would you represent this quantity then? [referring to a group of doozle cars]
Lucy:	I would probably do, um, **.
Instructor:	Ok, any others? Does your brain hurt yet?
Many students:	Yes!!
Scarlet:	I would do *~*.
Instructor:	You would do *~*. Ok, any others?
Kate:	^@
Instructor:	Ok. So let’s go back and think about some of these. If I were to use either of these 3, do you see a problem that might come up if I were to use any of these representations?
Hank:	You would need lots of them [symbols] to represent big numbers.
Joe:	You would need to know your operation – are we going to add or multiply or subtract?
Instructor:	So which one do you think makes the most sense now?
Hank:	*~
Instructor:	And why?
Hank:	It’s like a base 5, I guess, and then the one [symbol] on the left is saying how many 5’s you have and then the one [symbol] on the right would be saying how many 1’s you have. And then if you were to add another one [symbol] on the left you could do I guess 25’s.
Instructor:	So then what does tilde represent?
Hank:	The zero.

This discussion further reveals how the teachers are thinking about place value as they consider ways to symbolically represent doozle, the next number in the sequence. As before, some of the teachers are thinking about computations between the numerals as they represent the quantities symbolically rather than thinking about the location of the numeral within the symbolic notations. However, an interesting point noted by Joe is that they had never established that there was an operation between the symbols. Ultimately, Hank, like Lucy in the earlier discussion, draws on his prior knowledge of place value in the Arabic number system and convinces himself that the correct symbolic representation for doozle is **. In addition, we gain insight into another understanding that Hank possesses about place value as he relates the ten-to-one ratio seen between the place values in the Arabic number system to the five-to-one relationship in Orpda and mentions that a third place value would represent the number of groups of 25. Overall, both discussions reveal that some of the teachers are already drawing on their prior knowledge of place value and finding connections between the Orpda and Arabic number systems in order to gain a better understanding.

Once children start to gain additional number sense and become more familiar with the language and symbols associated with the base-ten number system, they are introduced to a one hundreds chart (see Figure 4.2). The purpose of having the children look at the chart is to help them begin to recognize patterns in the symbols used to represent different quantities. For example, children might recognize that each row and column ends with the same symbol, as well as the fact that each quantity on the diagonal, beginning with 11, is represented using the same symbol twice.

In order to help the teachers begin to recognize similar patterns in the symbols used to represent quantities in the Orpda number system, the instructor gave the teachers an @ skoobrat chart to complete for homework (see Figure 4.3). Since the base of the Orpda number system is flub (five) the @ skoobrat chart has five quantities per row rather than the ten quantities of the base-ten hundreds chart. Therefore, the first grouping size in Orpda is flub, which consists of flub (five) individual units. Skoobrat is the second grouping size which consists of flub flubs. The figure below provides an illustration of these grouping sizes using Orpda blocks.

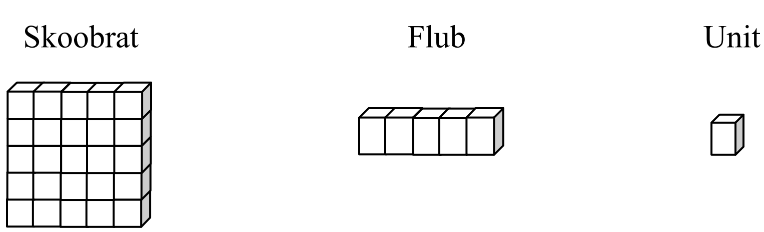


Figure 4.1. Orpda Blocks

One Hundreds Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Figure 4.2. One Hundreds Chart Used in Elementary Classrooms

@ skoobrat Chart

star *****	**at** **@**	**pound** **#**	**caret** **^**	flub *~
doozle **	sholt *@	pouflube *#	**carflube** ***^**	atty @~
atty-star @*	atty-at @@	atty-pound @#	atty-caret @^	poundy #~
poundy-star #*	poundy-at #@	poundy-pound ##	poundy-caret #^	carety ^~
carety-star **^***	carety-at ^@	carety-pound ^#	carety-caret ^^	star skoobrat *~~
skoobrat star *~*	skoobrat at *~@	skoobrat pound *~#	skoobrat caret *~^	skoobrat flub **~
skoobrat doozle ***	skoobrat sholt **@	skoobrat pouflube **#	**skoobrat carflube** ****^**	skoobrat atty *@~
skoobrat atty-star *@*	skoobrat atty-at *@@	skoobrat atty-pound *@#	skoobrat atty-caret *@^	skoobrat poundy *#^
skoobrat poundy-star *#*	**skoobrat poundy-at** ***#@**	skoobrat poundy-pound *##	skoobrat poundy-caret *#^	skoobrat carety *^~
skoobrat carety-star *^*	skoobrat carety-at *^@	skoobrat carety-pound *^#	skoobrat carety-caret *^^	at skoobrat @~~

* Note: Bolded entries were the symbols given to the teachers to help them as they filled in the symbols in the non-bolded entries on their own.

Figure 4.3. Completed Version of the @ Skoobrat Chart

Consequently, an @ skoobrat chart requires the teachers to count up to skoobrat and then beyond to @ skoobrat. Unlike with the one hundreds chart, the word names for each quantity were also included in the @ skoobrat chart. The reason to include the language along with the symbols was to help the teachers continue to become familiar with the new language of the Orpda number system. In addition, this would provide the teachers with an opportunity to not only recognize patterns in the symbols, but also in the language used to represent each quantity. The teachers were encouraged to think about how this language related to the language used to represent quantities in the base-ten number system. Finally, the chart was also used to help the teachers further their understanding of the importance of the location of a symbol in a number and how the position of a symbol in a number determines its value. More data collected from the teachers' reflections on how the @ skoobrat chart compared to the one hundreds chart will be provided as part of the data analysis for the third research sub-question.

<u>Tuesday: Recognizing the Base of a Number System and Unitizing</u>

The base of a number system is the number of symbols used in the number system to represent the sizes of the fundamental sets. The Arabic number system is considered to be a base-ten number system since it uses ten fundamental symbols, namely 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Different arrangements of these ten symbols can be used to represent higher quantities. The Orpda number system is a base-five number system, as it uses only five fundamental symbols which are *, @, #, ^, and ~. Arrangements of these symbols can then be used to represent higher quantities, as seen in the @ skoobrat chart. Consequently, the Arabic and Orpda number systems are considered to be efficient number systems. An efficient number system is one that consists of a limited number of symbols so that less memorization is required to understand and work with the number system. In addition, efficient number systems are made up of one grouping size, which is the base of the number system, and they have the ability to express an infinite number of higher quantities due to the patterns and locations within a numeral. It was important that Orpda be an efficient number system so that the teachers would hopefully spend less time trying to memorizing names and symbols, but instead would focus on recognizing the inherent patterns and understanding the importance of location within a numeral.

The act of unitizing involves children seeing ten as ten single objects or as one unit of ten. Similarly, children must also be able to recognize that ten groups of ten make one hundred. Children have difficulty developing the concepts associated with unitizing and part of that lack of understanding comes from not recognizing the ten-to-one relationship between the value of numerals representing a quantity (Cobb & Wheatley, 1988).

In order to work successfully in Orpda, the teachers also had to understand that the base of the Orpda number system was flub (five), that the relationship between the numerals representing a quantity was five-to-one, and then visualize the different grouping sizes inherent in the Orpda number system. The three activities used on the second day of Orpda were designed to encourage teachers to think about these ideas and the importance of unitizing when teaching their elementary students about place value. Upon completion of each activity, the teachers were asked to complete reflection sheets, and, in particular, note any "Aha!" moments they had while working through the activity. A copy of the reflection sheet that the teachers used can be found on the following page. The major points of the teachers' "Aha!" moments were then categorized to gain a better understanding of what each activity revealed about the teachers' overall understanding of place value.

Activity Reflection Guide

Name of Activity	Where's the math?	Guiding questions	Possible difficulties (for you or your students)	What triggered the AHA moment for you?

Figure 4.4. Activity Reflection Guide

How Many in All?

The first activity that the teachers completed during their second day of Orpda was called "How Many in All?" For this activity, the teachers worked in pairs, and they were given an envelope of cards, each card containing one of the five Orpda symbols, namely *, @, ^, #, or ~. Along with the cards the teachers received a six-sided die containing the same five Orpda symbols, along with the symbolic notation for flub, namely *~.

One teacher in the pair reached into the envelope and pulled out a card and placed the corresponding quantity of counters into a cup. Then, the second teacher rolled the Orpda die and placed the corresponding number of counters beside of the cup. The teachers worked together to determine, in Orpda, how many counters they had in all and then used Orpda notation to record the sum on their worksheet. A copy of the worksheet the teachers used can be found in Figure 4.5 on the following page. The teachers then repeated the process several times to form different quantities and recorded their results.

After analyzing each teacher's "Aha!" moment for this activity based on their reflection guides, I found that six teachers mentioned that grouping by flubs and unitizing helped them the most as they worked through this activity. On the other hand, three teachers noted that they were still trying to memorize word names and symbols as they participated in the activity, and did not consider unitizing. During the activity, some of the teachers drew a card that contained the symbol ~, corresponding to the quantity zero, and struggled initially to understand what quantity corresponded to this symbol. Two of the teachers noted that making the connection of ~ to the quantity zero helped them to best understand the activity as a whole. Some of the teachers referred to more than one topic in their reflections and were therefore placed into more than one

How Many in All?

Game for two. First player chooses a card from the envelope and places the indicated number of counters in the cup. The card is placed next to the cup as a reminder of how many are in the cup. The second player rolls the number cube and places that many counters next to the cup. Together they determine how many counters in all.

In Cup	On Side	In ALL

Figure 4.5. Activity Worksheet for How Many in All?

category. Furthermore, some teachers noted that they were still struggling with Orpda upon completion of the activity, and,consequently, did not have an "Aha!" moment to discuss.

How Many?

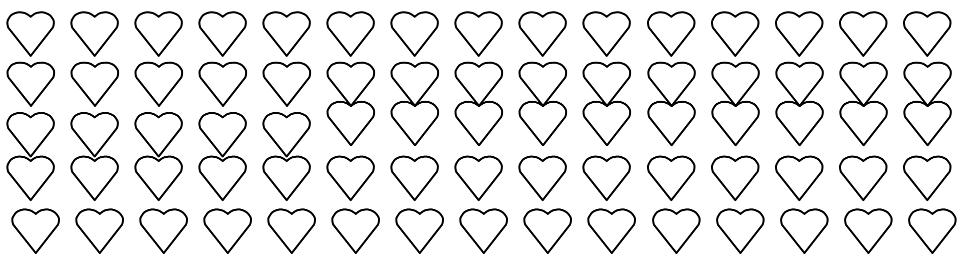
The second activity that the teachers completed titled "How Many?" involved showing the teachers a large number of objects drawn on a piece of paper along with a quantity written in Orpda notation. A copy of the worksheet given to the teachers for this activity is provided on the following page. The teachers were asked to circle the number of objects to represent the given quantity. Some of the problems had the teachers circling quantities indicated by two-digit numbers while others involved three-digit quantities. The instructor purposefully grouped flub objects in each column to continue to encourage the teachers to think about grouping by flubs, the base of the Orpda number system. However some teachers circled complete rows rather than columns of flub objects.

All of the teachers noted some "Aha!" moment that helped them associate the number of objects to circle with the quantity provided. Ten of the teachers again mentioned the importance of unitizing and grouping by flubs to help them know how many objects to circle for each problem. However, there were still three teachers that mentioned relying on memorization of the word names and symbols to help them circle the correct number of objects. Interestingly, some of the teachers that mentioned trying to focus on memorizing during the first activity, now started to recognize that grouping and unitizing helped them better understand the Orpda number system. Liz, one of the three teachers that mentioned memorizing the names and symbols as her way to work through the first activity noted how much she realized the importance of recognizing groups of flub to better understand how many objects to circle.

How many?

Circle ##

Circle *@



Circle *@~

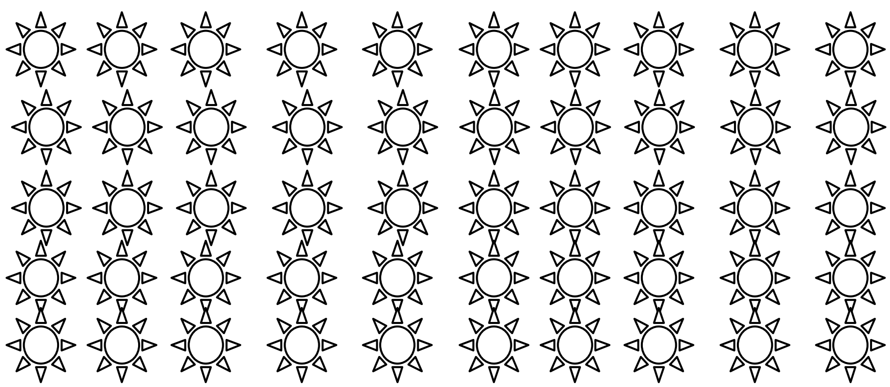


Figure 4.6. Activity Worksheet for How Many?

Liz: Grouping using the base *~ (5) helped to think in Orpda, also made memorizing the number symbols less important. I had a system as how to solve the "value problem."

Kate, on the other hand, provides the following "Aha!" moment:

Kate: During this activity, I was still struggling to remember the symbol values and was not noticing the ties between place value and amount.

Many of the other teachers revealed similar "Aha!" moments to those of Liz and Kate after completing this activity. The teachers that were beginning to recognize place values and grouping by flubs were starting to better understand Orpda, while those focused on simply memorizing all of the number names and symbols were having a hard time moving forward.

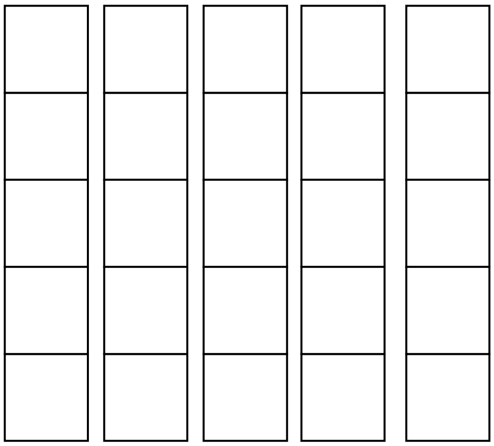
Fill the Flub Frames

The final activity that the teachers completed during the second day of Orpda was called "Fill the Flub Frames." Similar to the previous two, this activity is designed to continue to help the teachers to think about unitizing and the base of the Orpda number system. For this activity, the teachers were given several empty flub frames, similar to ten frames that would be used in a traditional elementary classroom, and were asked to collect a specific number of counters, the quantity given in Orpda notation. A copy of the worksheet the teachers used for this activity is provided in Figure 4.7 on the following page.

The teachers were then asked to use the counters to fill the flub frames and record how many flubs and units they created. This activity would be similar to having children in an elementary classroom use counters to fill in groups of ten and then record how many tens and units they create from a specific quantity of base-ten blocks. While this activity is very simple to complete, the teachers' "Aha!" moments were very revealing about how they were thinking

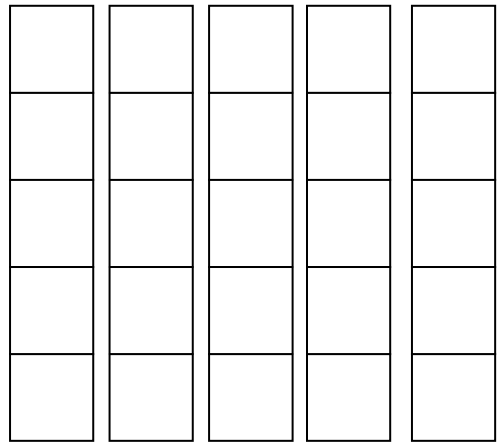
Fill the flub-frames

Get @# counters. Fill the flub frames.



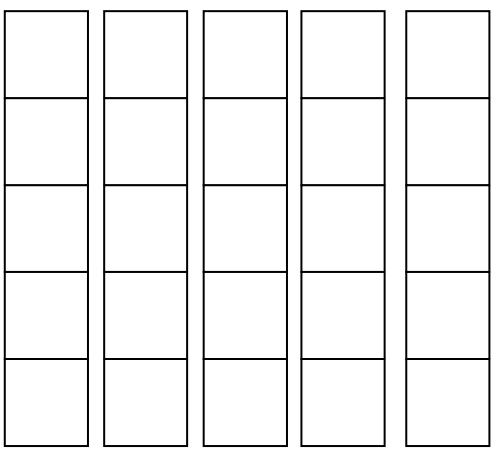
How many flubs? _______________
How many units? _______________

Get ^~ counters. Fill the flub frames.



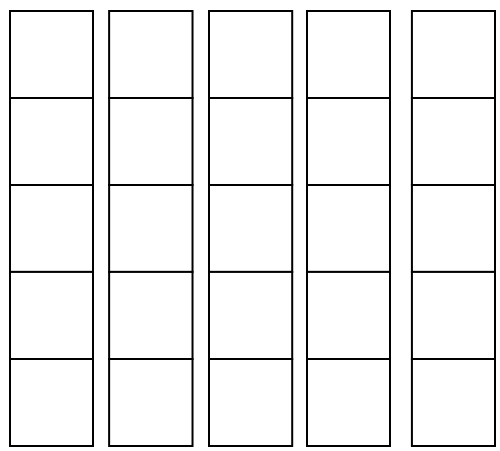
How many flubs? _______________
How many units? _______________

Get *@ counters. Fill the flub frames.



How many flubs? _______________
How many units? _______________

Get #^ counters. Fill the flub frames.



How many flubs? _______________
How many units? _______________

Figure 4.7. Activity Worksheet for Fill the Flub Frames

about the base of the Orpda number system and how the location of the symbol is important when writing numerals.

Ten of the teachers, nine of whom were included in the group from the previous activity, made reference to location as they mentioned how much this activity helped them understand that the location of a symbol in a numeral tells them how many grouping sizes are represented in a quantity. Kate, one teacher who was struggling to work with Orpda during the first two activities, provided a revealing reflection about how her knowledge of writing numerals, the location of the symbols, and the value of the number was challenged during this activity.

> Kate: I noticed that the number in the flubs place represented how many flubs and the number in the units place represented the number of units. When I was looking at the flub frames, it was the first time I made the connection between the names and values instead of just trying to memorize them.

In addition to understanding the meanings of place values within multi-digit numbers, two of the teachers mentioned noticing patterns within the activity and eventually being able to complete the remaining problems without needing to use the counters. Scarlet alluded to this fact in her reflection noting that her "Aha!" moment came from, "after completing the second problem, I saw the pattern and didn't need the blocks anymore." Overall, this activity provided the most insightful and revealing "Aha!" moments from the teachers as they began to understand place value on a deeper level. Upon completion of the second day of Orpda, the teachers' comments reveal how they are beginning to connect the location of a symbol to its value, visualize the grouping sizes, and recognize the importance of unitizing when teaching young students about place value.

<u>Wednesday: Unitizing, Regrouping and Number Relationships</u>

Counting Bags

The “Counting Bags” activity was the first activity completed by the teachers on the third day of Orpda. This activity continued to help the teachers associate the word names and symbols of different quantities within the Orpda number system, as well as recognize the role that unitizing plays when expressing a quantity symbolically. These goals are similar to the goals of the digit-correspondence tasks developed by Ross (1999) to help children move flexibly between different group sizes. For this activity, the teachers were given bags containing different quantities of various objects. The teachers were asked to count the number of objects in each bag, in Orpda, and record the quantity of objects noting the number of flubs and units created, as well as the symbol for the quantity along with the corresponding number name in the Orpda number system. A copy of the worksheet the teachers used to record their results for this activity is found on the following page.

Eight of the teachers referenced that their “Aha!” moments from this activity came from continuing to understand that the base of the Orpda number system determines the grouping size and unitizing helped them know the correct symbolic notation to use when expressing a quantity of objects. Tullula, had a particularly revealing reflection as she noted that “counting by flubs made it [the activity] much easier, it clicked!” While Tullula had mentioned recognizing the need to unitize in previous activities, her reflection reveals how this activity solidified her thoughts about unitizing and the importance of recognizing the base of a number system. Four of the teachers mentioned recognizing connections between the symbolic notation for the quantities of objects and their corresponding number names within the Orpda number system, suggesting their recognition of the importance of language patterns in the teaching of place value. One teacher reverted back to trying to memorize the names of numbers and their corresponding symbols within Orpda while working through this activity.

Counting Bags

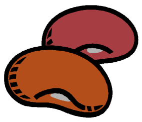
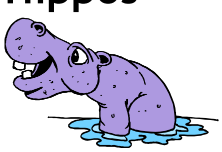
Bag of	Flubs	Units	Symbol	Words
Beans 				
Kangaroos 				
Rubber bands 				
Nails 				
Hippos 				
Mystery 				

Figure 4.8. Activity Worksheet for Counting Bags

Regrouping and Number Relationships

The final major points related to understanding place value that were emphasized through activities were related to regrouping and recognizing patterns and number relationships within the base-ten and Orpda number systems. Once children develop the ability to unitize, they can then begin to understand how the same number can be represented in different ways using regrouping. For example, the number 157 can be represented as one hundred, five tens, and seven units, but could also be represented as 15 tens and 7 units. In order to be able to regroup, children must understand the base of the number system as this determines the relationship between different grouping sizes. In the base-ten number system, ten groups of a particular size must be collected in order to make the next grouping size, as in moving from tens to hundreds. Similarly, in Orpda, five groups of a particular size must be collected in order to move to the next grouping size, as in moving from flubs to skoobrats.

The teachers completed two activities designed to foster this idea of regrouping and number relationships during the third day of Orpda. These activities were Race for a Flat and How Many Ways, which are described below. While working through these activities, the teachers were still encouraged to reflect on their understanding about place value as well as notice patterns within the Orpda number system that could be related to the base-ten number system.

Race for a Flat

This activity paired teachers together to play a game called "Race for a Flat." On a given turn, a teacher rolled a pair of dice with Orpda symbols written on them and found the sum between the two numbers he or she rolled. Then, the teacher collected the corresponding number of pre-grouped Orpda blocks to represent the quantity rolled. These blocks were placed in the appropriate spot on the place value mat. The teacher rolled the dice again, collected Orpda blocks to represent the sum rolled, and then added this sum to the quantity already on the place value mat. Sometimes this would involve trading units for flubs. The teachers worked in pairs alternating rolls of the dice, collecting the appropriate number of Orpda blocks and adding them to their place value mats, trading units for flubs, as needed. The first teacher to collect flub flubs and trade the flubs for a skoobrat won the game. In an elementary classroom, this game would be played using base-ten blocks consisting of units, longs (ten units grouped together), and flats (ten longs grouped together to create one hundred). This was the first activity that the teachers did that involved trading up to create the next place value, eventually ending with a three-digit number.

While observing the teachers work through this activity, I was looking to see how the teachers traded for different group sizes. Some of the teachers struggled at points when they rolled an amount that required them to trade for a different group size. For example, if a teacher already had # (four) units and then rolled @ (two), the teacher had to collect @ (two) individual units for a sum of ** (six) and then trade *~ (five) units for a flub with an additional unit left over. On the other hand, other teachers were able to visualize which Orpda blocks would represent the new quantity and skipped the trading step. This formative assessment continued to show the various levels of understanding the teachers had reached at this point during the week.

Upon completion of this activity, five of the teachers mentioned that understanding how to regroup units for flubs and flubs for skoobrats helped them the most as they played the game. Brad, one of the teachers in this group, noted that "recognizing how to add helped to speed up the game. For example, I had # and rolled a ^, so I traded for a *~ [flub] and @ units." Two teachers referenced the importance of recognizing how the location of a numeral in a quantity determines how many groups of a particular size are represented, while two other teachers again mentioned the importance of unitizing in this activity. Finally, one teacher, Hank, reflected on the importance of relating number relationships inherent to the base-ten number system with similar relationships found in the Orpda number system. He noted that "recognizing the relationships between flubs and groups of ten as well as skoobrats and groups of one hundred" helped him to easily trade throughout the game.

In addition to mentioning specific topics that helped them better understand place value during this activity, five of the teachers also mentioned gaining confidence in their overall understanding of Orpda during this activity. Quotes from two of the teachers, Tullula and Kate, given below help to further illustrate this fact.

> Kate: Once I got used to the idea of trading units for flubs, I became faster and more confident in playing the game. The more practice I had the better I became.
>
> Tullula: Finally figuring out how many stars [units] are in a flub and how many flubs are in a skoobrat…it's [Orpda] becoming automatic.

The activity, "Race for a Flat" revealed increased understanding among the teachers of regrouping as an essential component to learning about place value. The game also helped the teachers gain confidence in their overall understanding about this important area of mathematics.

How Many Ways

The final activity that the teachers completed within Orpda was called "How Many Ways." The purpose of this activity was to continue to encourage the teachers to think about regrouping and number relationships as important components to understanding place value. For this activity, the teachers were asked to use pre-grouped Orpda blocks to create the number *@#. This number would be similar to the number 123 in the base-ten number system. Then, the teachers were asked to use the blocks to show the same number in a different way. For example, the number 123 can be shown using one flat (hundred), 2 longs (groups of ten), and 3 units. However, the same number can also be shown using 123 units. To complete the activity, the teachers had to create all of the possible ways to represent *@# using flats (skoobrats), longs (flubs), and units. The teachers were encouraged to think about how they were figuring out the correct number of representations as they worked through the activity.

The teachers focused on two main topics as they considered this activity, namely regrouping and patterns/relationships. Some teachers mentioned both topics in their reflections and have therefore been included in both groups. For example, Tom mentions both topics as he notes that he "used the flubs to create a pattern and worked off of the flubs to get to the next level [place value]." Many of the teachers created an organized list to help them keep track of the number of different representations they came up with and mentioned the importance of their list in helping them work through the activity and notice the inherent patterns and number relationships.

<u>Thursday: Addition and Subtraction</u>

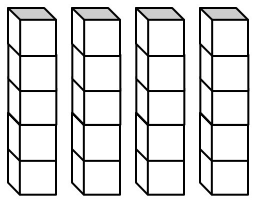
On Thursday, the focus of the class turned towards working problems that involved addition and subtraction in Orpda. As with each of the previous three days, manipulatives were available at each table for the teachers to use to help them solve the problems. The teachers were presented with the first word problem to solve.

> Problem: Sarah has sholt dolls and her friend Rachel gave her some more dolls. Now Sarah has carety dolls. How many dolls did her friend give her?

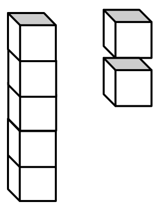
The teachers were given some time to work on the problem independently. During this time, the instructor and I walked around the room looking to see how each teacher went about solving the problem. Lucy was the only teacher using the manipulatives. The remaining teachers were referring to their completed at-skoobrat charts or using more traditional algorithms to add and subtract multi-digit numbers.

The instructor asked Lucy to demonstrate how she solved the problem for the class. The following is the resulting explanation, as recorded in my field notes.

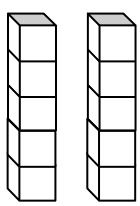
Lucy: I knew I had to get to this point (carety).



Lucy: I had sholt dolls which is this many.



Lucy: So, I added pound units because I knew I needed a complete flub and that would give me at flubs.

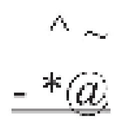


Lucy: Then, I needed at more flubs to make carety, which is caret flubs. So, the answer is that Rachel gave her atty-pound dolls.

Analysis of Lucy's explanation reveals that she had clearly recognized the importance of unitizing as she immediately considered how many additional units she needed to make an additional flub from what she had. In addition, Lucy's use of the manipulatives to help her solve the problem reveals that she still needed to make concrete models in order to illustrate the situation and see how to go about solving the problem.

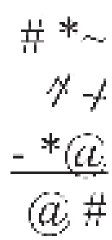
Since Lucy was the only teacher that used the manipulatives to solve the problem, the instructor then asked the remaining teachers, “How did you think about the problem if you did not use the manipulatives?” Betty, one of the high school teachers sitting in the back of the room, asked if she could come to the board and show how she solved the problem. Betty proceeded to illustrate how she used symbols to solve the problem.

Betty: We did it like this. We had carety minus sholt.



Betty: So, we borrowed a flub from carety to get this.

Betty: Then, we did flub minus at to get pound and pound minus star gives you at.



Analysis of Betty’s solution shows that she has reached a higher level of abstraction as compared to Lucy as she was able to use symbols to help her solve the problem. Interestingly, all four of the high school teachers at Betty’s table used this approach which seems to suggest that they are more used to using the traditional algorithms to solve addition and subtraction problems as compared to using manipulatives.

The teachers were presented with another similar word problem to solve. Many more of the teachers used manipulatives to solve this problem. Lauren, an elementary teacher, came to the front of the room to demonstrate how she solved the problem, similar to what Lucy had done for the first problem. After Lauren finished, the instructor mentioned that she noticed that many more of the teachers had used the manipulatives this time to solve the problem. She then asked, "So did the manipulatives help you or hinder you?" Many of the teachers responded with "Help!" The instructor then asked, "Would anyone say that they hindered them?" None of the teachers responded.

While the teachers were not asked to complete reflections on the word problem activities, their discussions and thoughts regarding the activities reveals some interesting thoughts about how they were thinking about solving problems and using manipulatives. The group of high school teachers naturally gravitated towards using a traditional algorithm to solve the word problems rather than using manipulatives. This might suggest that teachers of higher content areas are more focused on algorithms as they use them more often to solve problems in their own teaching. However, Lucy, the one teacher that used manipulatives to solve the first problem, is a college teacher. Thus, the choice of a particular solution method may not be associated with the level of content for the teacher. At the conclusion of the class, all of the teachers recognized how much the manipulatives helped them see how to solve a problem, despite the fact that they did not initially use them. During the discussion, Betty added, "I did the problems in my head first, and then went back and checked them with the manipulatives. I now think it would have been much easier to do them with the manipulatives first, then go back and check them in my head."

Since this was the final activity that the teachers completed in Orpda for the week, to summarize, the instructor asked the teachers, "What helped you understand the Orpda number system?" The following discussion resulted as many teachers contributed ideas regarding what most helped them throughout the week.

Hank: I think visually seeing how the units make up a flub with actual manipulatives and also with the flub frames.

Joe: I had to learn the terminology and then using these manipulatives helped me on the addition. We [teachers at the high school table] went straight to subtraction and addition and then when we got all the answers we went back and used the manipulatives because to us that's a little more difficult. We're trying to take the derivative and children are trying to add 2 and 2. I liked these manipulatives once I started using them.

Scarlet: I think being able to link Orpda to base-ten and already knowing there was a limited number of symbols and seeing the patterns.

Again, Hank and Joe, two high school teachers, mentioned how much they realized that the manipulatives helped them work a problem, despite the fact that they were not used to using them in their own teaching. Scarlet also revealed that she was drawing on her prior knowledge of place value in the base-ten number system to help her better understand Orpda.

The instructor then asked the teachers to also consider areas where they struggled throughout the week. The teachers' contributions to this discussion are given below.

Hank: I think the hardest part for me was remembering the order of the symbols, especially # and ^. I kept getting mixed up. So, if you are not comfortable with these symbols that makes dealing with the larger numbers harder.

Lucy: Likewise, I got really hung up on doozle and sholt and I still do.

Tom: When you had to jump a level, a flub, like when we were counting from star to atty. If you had us doing that in subtraction and addition, that made it tough.

Kate: I had trouble thinking in the Orpda number system and staying in the Orpda number system instead of converting.

The first three teachers' comments above reveal that they struggled in the same areas where children struggle with understanding place value in the base-ten number system. This shows that Orpda has helped the teachers recognize those struggles that children are going to have as they try to understand place value. Kate's comment relates closely to similar conclusions that have been found with using alternate base systems with teachers in that the teachers try to convert to base-ten rather than working with the new base.

Friday: Invented Algorithms for Addition and Subtraction

The final day of the week focused on recognizing invented algorithms that children use to solve addition and subtraction problems. While the activities done on Friday did not use Orpda, the purpose of having the teachers consider invented algorithms was to help them recognize the importance of a strong understanding of place value in relation to performing computations. The instructor opened the class by having the teachers mentally solve the problem, 461 + 296, without using any paper and pencil.

After allowing the teachers to have time to think about their answers, the instructor then had the teachers explain how they solved the problem. Some of the teachers mentioned using the traditional algorithm of working from right to left, carrying as needed. Other teachers used invented algorithms to help them solve the problem more easily in their heads. For example, Joe used the invented algorithm of adding the numbers according to their place values; that is, he added 400 + 200, then 60 + 90, and then 1 + 6, to arrive at his answer of 757. The instructor then had the teachers watch a video demonstrating various other invented algorithms that children used to solve the same problem. A similar approach was used to discuss invented algorithms for subtracting multi-digit numbers.

Once the teachers were familiar with the various invented algorithms for addition and subtraction, the instructor concluded the class by asking the teachers, "What are the advantages to allowing children to use invented algorithms?" Nearly all of the teachers contributed to the discussion mentioning reasons such as, "They make more sense to the student," "They allow the student to grasp the concept better and are longer lasting," and "They allow children to take ownership of their learning." Kate added that, "because many of the invented algorithms involve using knowledge about place value, they can show the teacher what the children know about place value based on how they use the algorithm."

At the end of the week of Orpda, the teachers were asked to respond to a final discussion board question. The question was the following: Consider the entire week of activities related to Orpda. In what ways did your personal strengths, weaknesses, beliefs, and/or dispositions about teaching and learning mathematics (or about Orpda) enhance or inhibit your participation? The purpose for asking the teachers this question was to have them reflect upon their initial reactions to Orpda, their thoughts about the Orpda activities, along with their thinking related to teaching place value once the week was complete. The following quotes from a selected sample of the teachers reflect the overall feelings of the group as a whole as they relate to the teachers' thoughts about the Orpda activities and how the activities challenged their beliefs and knowledge about place value. Melissa, a prospective elementary teacher with a medium level of math anxiety notes,

> Melissa: To be honest, at first I thought we were kind of dragging the whole Orpda thing out. But by the third day when we had been doing so many activities, I kind of 'got it.'

Kate, another prospective elementary teacher with a low level of math anxiety that continually reflected on her understanding of place value throughout the entire week provides her reflection on a particular activity.

> Kate: My ultimate 'Aha!' moment occurred during the Fill the Flub Frames activity. Honestly, at first I thought it was pointless because I knew what the answers were, but it's not always just about knowing the right answer. It's about understanding

> the process. That activity was when I made the connection between the amounts, symbols, and their names. Once I made this connection I was able to participate more confidently in our discussions about Orpda.

The final two quotes come from Brad and Tom, two teachers certified to teach secondary mathematics with experience teaching both at the high school and college levels. Both teachers earned an undergraduate degree in mathematics and possessed a strong mathematics background entering the class.

> Brad: As we started the Orpda and place value unit, I admit I was a bit skeptical. Being a secondary certified teacher, I don't really spend much thought on place value. I wondered why so much emphasis on the topic. As the week progressed, however, I started to realize why a good understanding of place value is an essential building block on which much of math is laid. Working in Orpda caused me to be challenged and really think about and see where common mistakes are made in place value…I never really placed much emphasis on the discovery process before this class. I also see it as an important tool for learning math now.

> Tom: Brad, as a secondary math teacher as well, I also had to adjust with the place-value concept. The question that came to my mind that you hinted at was: What if my current students had been taught place-value at an early age using much the same methods and approaches that we used for Orpda? Would I be teaching students with completely different mathematical minds? I don't know, but my bet is that I would be. Just a thought.

Summary

Analysis of the teachers' "Aha!" moments upon the completion of each Orpda activity reveals that the teachers noticed three critical components necessary for developing a conceptual understanding of place value, namely unitizing, regrouping, and recognizing the meaning of different place values within a multi-digit number. Among the 72 "Aha!" moments that were analyzed for each of the Orpda activities, 27 of the teachers' reflections mentioned ideas related to unitizing. In particular, the second activity that the teachers did on Tuesday, "How Many?," really helped the teachers begin to recognize the importance of unitizing. This activity involved having the teachers circle the number of objects that represented a specific quantity. The instructor purposefully grouped the objects together such that there were flub objects in each column. While not all of the teachers recognized this initially as they circled the objects for each problem, those teachers that did make this connection mentioned how much it helped them complete the activity in their reflections. Tom, a high school teacher that had struggled with Orpda the first day, mentioned that "figuring out that the columns are flubs made it easier to know how many objects to circle." Melissa, an elementary teacher, also added that "once I got to the last problem, I noticed that it was easier to group instead of circling the objects star by star [one by one]."

In addition to unitizing, 14 of the teachers' "Aha!" moments were related to regrouping. Two of the activities that the teachers did on Wednesday, "Race for a Flat" and "How Many Ways," particularly emphasized the importance of understanding how numbers can be regrouped in order to complete the activity. During "Race for a Flat," the teachers continually traded units for flubs and then flubs for skoobrats in order to win the game. In the beginning of the game, many of the teachers had to collect individual units to represent the quantity they rolled and then trade the units for flubs one at a time. However, as they progressed through the game, they began to recognize how to regroup and became faster at trading for different group sizes. Brad made note of this fact in his "Aha!" moment as he mentioned, "I recognized how to speed up the game. For example, I had # units and I rolled a ^, so I immediately traded for a *~ and @ units rather than counting the units out one at a time." Lauren also mentioned how regrouping helped her complete the "How Many Ways" activity: "My 'Aha' moment came when I figured out a pattern for switching out stars for flubs."

Finally, eleven of the teachers' reflections on the activities were related to the importance of understanding the meaning of different place values within a number. The majority of the teachers' reflections related to location occurred after the "Fill the Flub Frames" activity. While this is considered to be a very simple activity, it brought out some of the teachers' most revealing "Aha!" moments. After completing the activity, Tom mentioned, "The pattern: 1st symbol = Flubs, 2nd symbol = Units (Now I don't have to use my counters!)" Eight other teachers provided reflections similar to Tom's for this activity.

At the end of the week, many of the teachers referred to how all of the activities helped them build a deeper understanding of place value. Cinderella commented on this fact noting, "This week reinforced my belief in students needing different and multiple exposures of concepts to fully understand. Doing all of the different tasks really helped me understand better." Melissa added, "I now see the importance of the whole unit. I see the benefit of the many different activities you can do with numbers and place value, and I see the value of really understanding place value."

Question 2

What do comparisons of pre- and post-Orpda concept maps reveal about teachers' understanding of place value concepts after experiencing Orpda?

Prior to the Monday morning class of Orpda, the teachers were asked to create a concept map that revealed how they thought about concepts related to place value. The teachers then participated in many different activities designed to deepen their understanding of place value as they worked with the Orpda number system. Four weeks later, at the end of the summer class, the teachers were asked to create a second concept map illustrating their understanding of place value. Both the pre- and post-Orpda concept maps were then analyzed to begin to gain a sense of how the teachers' thinking about place value changed after experiencing Orpda.

Analysis of the teachers' pre- and post-Orpda concept maps was conducted in three phases in order to gain a better understanding of how the teachers thought about place value and connected the related ideas together in their minds. The first phase of analysis involved categorizing the types of maps that each of the teachers drew both before and after Orpda, while the second phase of analysis focused on the concepts that the teachers included in their maps. The final phase of analysis was devoted to understanding how the teachers connected all of the different concepts together. The analysis of the data collected from each of these phases will be described in the three sections below. In addition, each section will discuss what the data from each phase of the analysis reveals about the teachers' understanding of place value concepts.

Phase I: Categorizing Teachers' Pre- and Post-Orpda Concept Maps

The first phase of analysis involved categorizing the teachers' concept maps as either spoke, net, or chain maps, as described by Kinchin and Hay (2000). Conceptual understanding is defined as knowledge that is rich in connections with many pathways connecting ideas and concepts. This suggests that a net concept map is considered to show the deepest level of conceptual understanding since it is characterized by many connections between all of the concepts within the map. Spoke and chain maps reveal less of a conceptual understanding (fewer connections), with chain maps falling at the bottom of the hierarchy. In a chain map, concepts are linked together like a chain. Therefore, the connection from a concept at the top of the chain to a concept at the bottom of the chain would require one to traverse all of the concepts in between. The only direct connections that exist are found between concepts at the same level of the chain or one level immediately above or below. Table 4.3 on the following page gives the categorization for each teacher's map constructed both before and after Orpda.

After categorizing the teachers' pre-Orpda concept maps, I noticed that three of the teachers created net maps, while eight of the teachers used a spoke style, and the remaining two chose to create a chain map. After Orpda, three of the teachers again chose to use a net map, although there were some differences among these three teachers as compared to the pre-Orpda maps. Again, spoke maps were among the majority with eight teachers, and only one teacher created a chain map. One teacher's post-Orpda concept map did not fit any of the three types and was therefore not categorized.

Comparing the pre-Orpda concept maps with the post-Orpda concept maps, I noticed that only three of the teachers, namely Liz, Hank, and Brad, created a different type of concept map after working in Orpda. Liz used a chain structure to connect place value concepts in her pre-Orpda concept map and then changed to a spoke structure for her post-Orpda concept map. Since concept maps that follow a spoke structure are thought of as showing a slightly deeper level of understanding as compared to a chain structure, it appears that Liz showed more

Table 4.3
Categorization of Teachers' Pre- and Post-Orpda Concept Maps

Name	**Teaching Experience**	**Pre-Orpda Concept Map**	**Post-Orpda Concept Map**
Liz	Pre-intern* *Elementary*	Chain	Spoke
Lauren	Pre-intern *Elementary*	Spoke	Spoke
Kate	Intern** *Elementary*	Spoke	Spoke
Melissa	Pre-intern *Elementary*	Spoke	Spoke
Tullula	Pre-intern *Elementary*	Spoke	Spoke
Scarlet	Intern *Elementary*	Spoke	Spoke
Cinderella	Intern *Middle*	Net	Net
Joe	2 years *High*	Spoke	None***
Betty	Intern *High*	Spoke	Spoke
Hank	Intern *High*	Net	Spoke
Tom	2 years (Alg, Geo) *High*	Chain	Chain
Brad	1+ years (Dev. Math) *College*	Spoke	Net
Lucy	1 + years (Dev. Math) *College*	Net	Net

** A pre-intern teacher is one that will be completing a one-year internship beginning in the fall semester. Therefore, a pre-intern teacher is not considered to have teaching experience.*

*** An intern teacher is one that had already completed a one-year internship at the time the data collected. Therefore, an intern teacher is considered to have one year of teaching experience.*

**** This teacher's post-Orpda concept map could not be categorized as any of the three types and did not fit the definition of a concept map.*

evidence of a conceptual understanding of place value after Orpda. A similar argument can be made for Brad, as he changed from a spoke structured concept map before Orpda to a net structure, the highest level of conceptual understanding, after Orpda. On the other hand, Hank showed evidence that he possessed less of a conceptual understanding after Orpda as compared to before since he constructed a net map and then changed to a map that followed a spoke structure. This is a very unusual result, as Hank was one of the most reflective teachers throughout the week of Orpda. He continually participated in class discussions and used appropriate terminology as he related Orpda to the base-ten number system. Analysis of other data sources would classify Hank as one of the teachers that showed evidence of possessing a deep level of understanding about place value.

Cinderella and Lucy connected topics related to place value together using a net map both before and after Orpda. This suggests that these two teachers maintained a deep level of understanding regarding place value concepts throughout the week of Orpda. However, just looking at the type of map created may not be the best way to analyze connections among concepts or to measure conceptual understanding.

Phase II: Focusing on the Concepts

The second phase of analysis considered the actual concepts related to place value that the teachers included in their pre- and post-Orpda concept maps. Categories for the concepts that the teachers referred to in their maps were created to encompass the teachers' use of different language to refer to the same idea. Each teacher's map was then coded using these categories. Appendix E includes a list of these categories along with their definitions and examples of phrases from the teachers' concept maps coded using these categories. Figure 4.9 on the following page shows the numbers of teachers that made reference to a particular category in both their pre- and post-Orpda concept maps.

Close analysis of Figure 4.9 reveals a few key differences between the concepts teachers included in their pre- and post-Orpda concept maps. The most apparent difference between the two maps can be seen for the topic related to patterns within the base-ten number system. Prior to working in Orpda, none of the teachers mentioned patterns in their pre-Orpda concept maps, while seven teachers mentioned ideas related to patterns in their post-Orpda concept maps. In addition, two of the teachers made reference to unitizing in their pre-Orpda concept maps compared to six in the post-Orpda concept maps. Unitizing was one of the four themes that the instructor and I had identified, prior to starting Orpda, as being important to understanding place value, as noted in the literature. A third key difference seen between the two maps is related to ideas of teaching place value. Before Orpda, four of the teachers made reference to topics associated with teaching place value. These references included tools for teaching place value such as manipulatives and place value charts as well as ways for assessing students' place value understanding. Upon completion of Orpda, eight made similar references to teaching place value in their post-Orpda concept maps. Finally, seven of the teachers considered different types of numbers as an important area related to place value understanding before Orpda compared to only two of teachers making similar references after Orpda.

Remembering that four weeks had passed from the conclusion of Orpda before the teachers completed their post-Orpda maps reveals that these differences are significant. During these four weeks, the teachers did not discuss place value concepts. Therefore, inclusion of a particular concept in the teachers' post-Orpda concept maps reveals that the teachers maintained

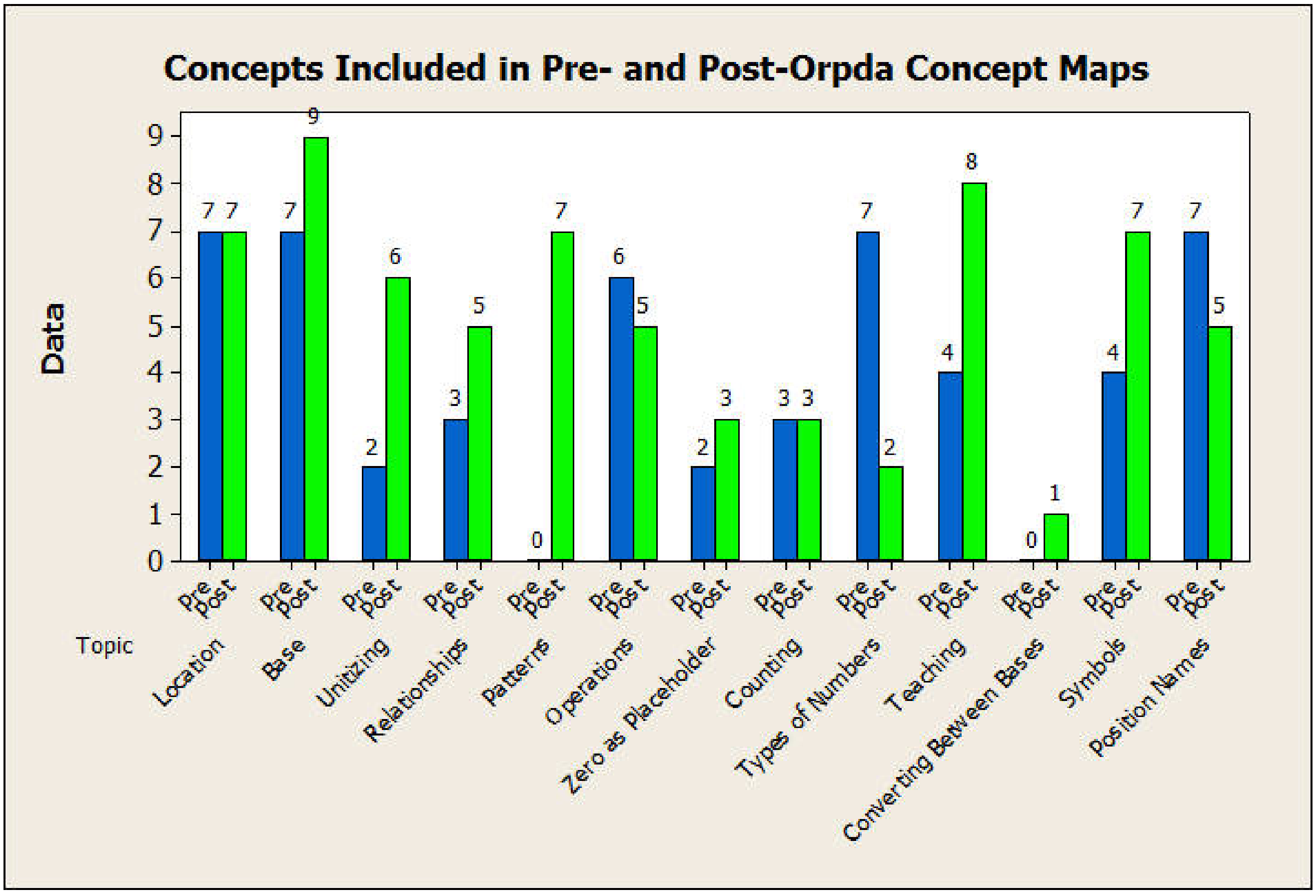


Figure 4.9. Concepts Included in Pre- and Post-Orpda Concept Maps

a realization of the importance of that concept to an overall understanding of place value throughout the entire summer course.

One interesting result from this figure is that none of the teachers mentioned converting between bases of different number systems in their pre-Orpda concept maps, while one teacher made reference to this idea in the post-Orpda concept map. A common criticism of using alternate bases to force teachers to reflect on their own understanding of place value is that the teachers convert problems to base-ten and then convert their answers back to the new base. As a result, teachers focus more on the conversion process rather than thinking about the actual place value concepts. It is important to note, however, that the teacher that included this category in his post-Orpda concept map had experience teaching higher-level mathematics. Consequently, he may not have been referring to converting between bases while working in Orpda, but could have been referencing the idea of using different base systems, a topic sometimes taught in the higher grades.

Overall, the teachers' pre-Orpda concept maps as a group showed a much more procedural understanding of place value. A concept map was considered to be procedural if it made extensive reference to topics including mathematical operations with numbers, such as addition and subtraction, as well as different types of numbers, such as fractions and decimals. However, the teachers' post-Orpda concept maps as a group reflected more of a conceptual understanding of place value. A concept map was considered to be conceptual if it made more references to ideas such as unitizing and grouping, patterns, location, and number relationships within the base-ten number system. Figure 4.9 reflects the increased numbers of teachers that mentioned more conceptual topics in their post-Orpda concept maps as compared to their pre-Orpda concept maps.

Phase III: Connecting Concepts Together

The final phase of analysis of the teachers' pre- and post-Orpda concept maps involved creating adjacency matrices to better understand the connections teachers made between the concepts they related to place value. Chapter three describes that an adjacency matrix is created from a concept map by first labeling all of the nodes in the map using alphabetical letters. These labels are then used to create the columns and rows of the matrix. Between any two labels, a value of 1 is entered into the corresponding entry of the adjacency matrix if these two concepts are connected by a line in the concept map. Otherwise, if no connection exists in the map between the two nodes corresponding to the labels, an entry of zero is entered into the matrix.

Squaring a matrix, just like squaring an integer, involves multiplying the matrix by itself. However, the process of squaring a matrix is different from the process of squaring an integer, such as 3. In order to square an integer, we multiply 3 times 3 to get 9. Squaring matrices involves multiplying a row of the first matrix, by a column of the second matrix. Then, the results are added to create the entry in the squared matrix. For example, consider the matrix given below.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Since matrix A has two columns and two rows, the squared matrix, A^2, will also contain two columns and two rows, or four entries. Each entry in A^2 will be computed by multiplying the entries in the particular row and column of A corresponding to the entry being computed in A^2. That is, to create the entry in the first row and first column of A^2, using matrix A above, we would multiply the first number in the first row of A by the first number in the first column of A, and then multiply the second number in the first row of A by the second number in the first column of A. Then, we would add the two results together to obtain the value in first row and first column entry of A^2. This process is outlined below.

$$A^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} (1\times 1)+(2\times 3) & (1\times 2)+(2\times 4) \\ (3\times 1)+(4\times 3) & (3\times 2)+(4\times 4) \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$

The purpose of squaring the adjacency matrices created from the teachers' pre- and post-Orpda concept maps was to better recognize the connections teachers made between the various concepts they included in their maps, since just identifying the type of concept map may not reveal conceptual understanding. While several of the teachers created maps of the same style (net, spoke, or chain), individually, their maps still looked very different and contained many different concepts and connections. Consequently, it was difficult to identify connections just from looking at a teachers' concept map and understand exactly what connections the teacher was making between the included concepts. Examining the squared adjacency matrices allowed the connections among concepts to become more apparent.

After forming and squaring the adjacency matrices created from each teacher's pre- and post-Orpda concept maps, two main characteristics of the matrices were considered. The numerical entries along the main diagonal of a squared adjacency matrix indicate how many other nodes in the map are directly connected to a particular node. Therefore, I first examined the entries along the main diagonal of each teacher's squared adjacency matrix that corresponded to nodes I had coded using the categories in appendix E. I compared the main diagonal entries from each teacher's pre-Orpda concept map with those of the post-Orpda concept map, looking for those entries with the highest numerical values. An entry with a high numerical value along the main diagonal corresponds to a category that the teacher recognizes as most important to understanding place value. Therefore, comparing the values on the main diagonal of the matrix helped me understand what topics the teachers considered important to understanding place value before Orpda as compared to after Orpda.

Tables 4.4 and 4.5 on the following two pages provide a list of the categories from appendix E developed in the second phase of the analysis. The entries in the table show how many connections that a coded node shares with the other nodes in the map for a particular teacher. Some teachers created “clumps” of nodes that were all coded as belonging to one of the particular categories. A common example of this occurred when teachers included operations in their concept map. Typically, teachers would include four nodes, namely addition, subtraction, multiplication, and division and draw lines connecting all of these nodes with one another. I would then circle the entire “clump” and code it as operations. In this case, the number of different connections that existed among all of the nodes in the “clump” was used to determine the entry in the table.

Tables 4.4 and 4.5 begin to reveal some differences in what categories teachers felt were most important to understanding place value before and after Orpda. First, looking for the highest entry in table 4.4 for each teacher reveals that the teachers considered five categories as being important to place value before Orpda. These categories were base, position names, operations, types of numbers, and teaching. Hank was the only teacher whose highest entry occurred for the category related to the base of a number system, and therefore, was the only teacher that considered the base of a number system as most important to understanding place value before working with Orpda. On the other hand, the categories of position names, operations, and teaching each showed up as being most important to place value for three of the teachers, while the remaining two teachers’ pre-Orpda concept maps were primarily focused on types of numbers.

Table 4.4

Number of Concepts Directly Connected to a Given Concept – Pre-Orpda Concept Map

Concept	***Tom***	***Lucy***	***Cinderella***	***Hank***	***Liz***	***Kate***	***Tullula***	***Betty***	***Scarlet***	***Lauren***	***Brad***	***Melissa***
Location		4	3	3			1	2		1		
Base	2	4	5	4				1			3	4
Unitizing								2		1		
Position Names (Ones, Tens, Hundreds, etc.)	6				2	4	8	6				4
Relationships				1							3	
Symbols			4	3						5	2	
Patterns												
Operations		3	6		3					1	6	5
Zero		2									2	2
Counting					2					1	1	
Types of Numbers	10				6	1			7	4		4
Teaching		6						2	12	9		
Converting Between Bases												

Table 4.5

Number of Concepts Directly Connected to a Given Concept – Post-Orpda Concept Map

Concept	***Tom***	***Lucy***	***Cinderella***	***Hank***	***Liz***	***Kate***	***Tullula***	***Betty***	***Scarlet***	***Lauren***	***Brad***	***Melissa***
Location		7	3	2	2	1	2			4		3
Base	7		3	1	2		3		1	4		
Unitizing		4		2		3		1		2	4	
Position Names (Ones, Tens, Hundreds, etc.)				3				1	1			7
Relationships	3		4	2					1		3	
Symbols	2		6	1		1		11	1		1	
Patterns	2	5	6		2			1			3	1
Operations		15	4							5	3	5
Zero	3	3									3	
Counting		5			2	2						
Types of Numbers	2										7	
Teaching	8					2	2		2	5		3
Converting Between Bases											2	

Again, this suggests that most of the teachers viewed place value from a more procedural perspective prior to working with Orpda as their concept maps focused on more procedural categories, such as operations and types of numbers. The category of position names only refers to the actual names of the positions of symbols in a multi-digit number, such as the one's place, ten's place, and hundred's place. A separate category, named location, was created to code those nodes that described the location of a symbol in a number, regardless of the position names. Therefore, for the majority of the teachers whose highest entries on the main diagonal did not occur in the categories of operations or types of numbers, their pre-Orpda concept maps were focused on including many nodes to reflect the different terms used to describe positions of symbols within numbers, but did not refer to the concept of location in general as being important to understanding place value.

Examining the highest entries on the main diagonal of each teacher's post-Orpda concept map, reflected in table 4.5, shows a different perspective of what categories the teachers considered most important to understanding place value. In this case, some teachers had their highest entries occur in more than one category, and were, therefore, counted in both categories. For example, Cinderella's highest main diagonal entry was 6 which occurred in both the symbols and patterns categories. Consequently, Cinderella was counted in both categories when tallying the total number of teachers that had their highest main diagonal entries appear in a particular category.

Table 4.5 reveals that all of the categories, except for relationships and zero, contained a highest main diagonal entry for at least one teacher. This suggests that, while the teachers all held a very similar view in regards to the most important concepts related to place value in their pre-Orpda concept maps, the teachers viewed place value differently from one another after

Orpda. The teachers were now beginning to recognize the importance of concepts such as base, symbols, and patterns, among others, as important to developing an understanding of place value. These concepts are considered to reflect a more conceptual understanding of place value.

Comparing each individual teacher's highest entries on the main diagonal in their pre- and post-Orpda concept maps also reveals some interesting differences. Cinderella, Brad, and Melissa all had their highest main diagonal entries appear in the category of operations for their pre-Orpda concept maps which suggested that they had a more procedural understanding of place value. However, in their post-Orpda concept maps, Cinderella's highest entries occurred in the categories of patterns and symbols, while Brad's appeared in types of numbers and Melissa's appeared in position names. This suggests that Cinderella's view of place value after Orpda was becoming more conceptual while Brad and Melissa continued to focus more on procedural concepts related to place value. Similarly, Tom and Liz primarily focused on types of numbers in their pre-Orpda concept maps, while their post-Orpda concept maps both focused on more conceptual categories. Tom's highest entry on the main diagonal in his post-Orpda concept map occurred in the category of teaching, while for Liz, the categories of location, base, patterns, and counting all contained her highest main diagonal entry. Copies of Liz's pre- and post-Orpda concept maps can be found on the following two pages. The major differences in her maps reveals a very different perspective of place value before Orpda compared to after Orpda.

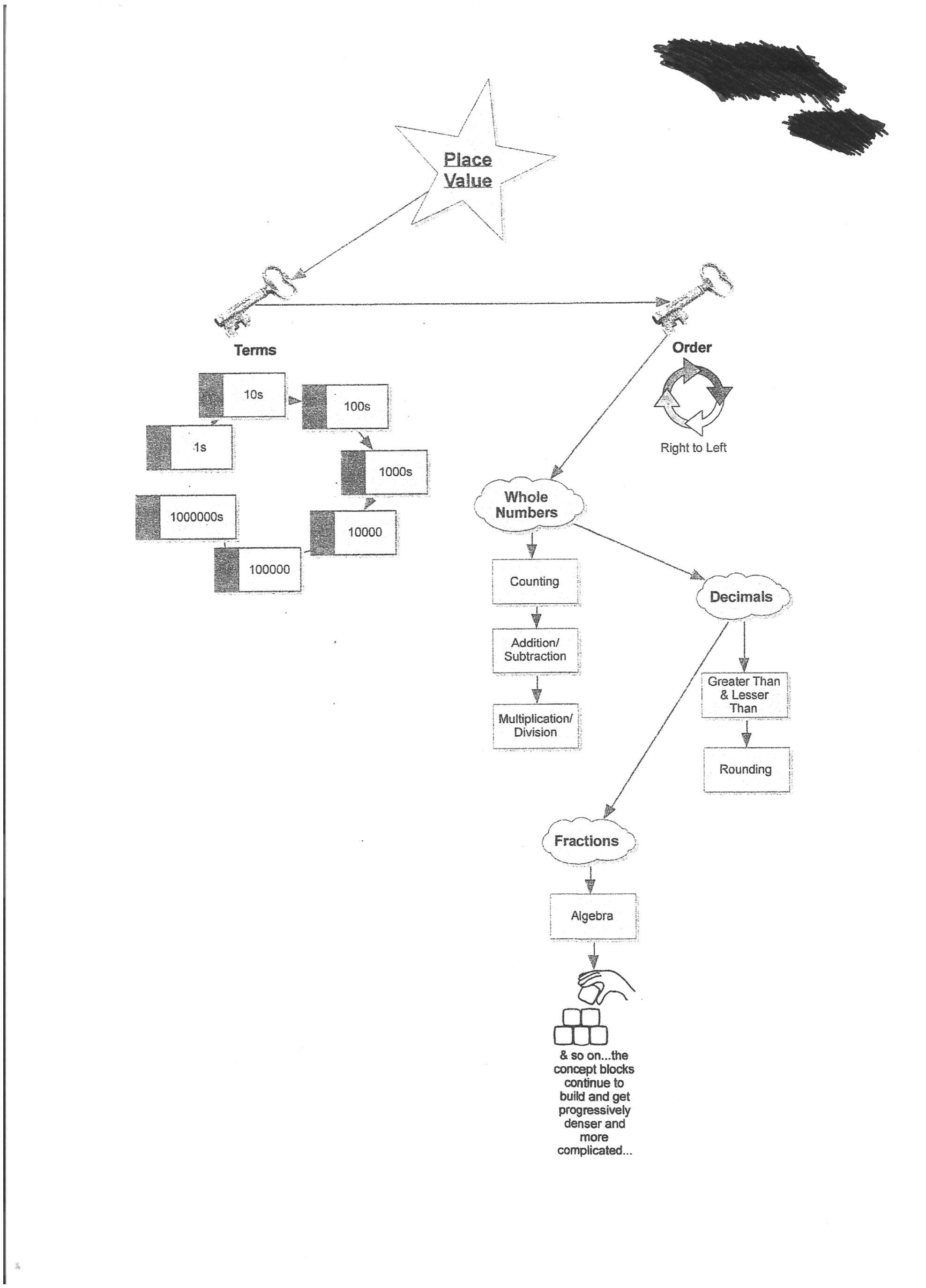


Figure 4.10. Liz's Pre-Orpda Concept Map

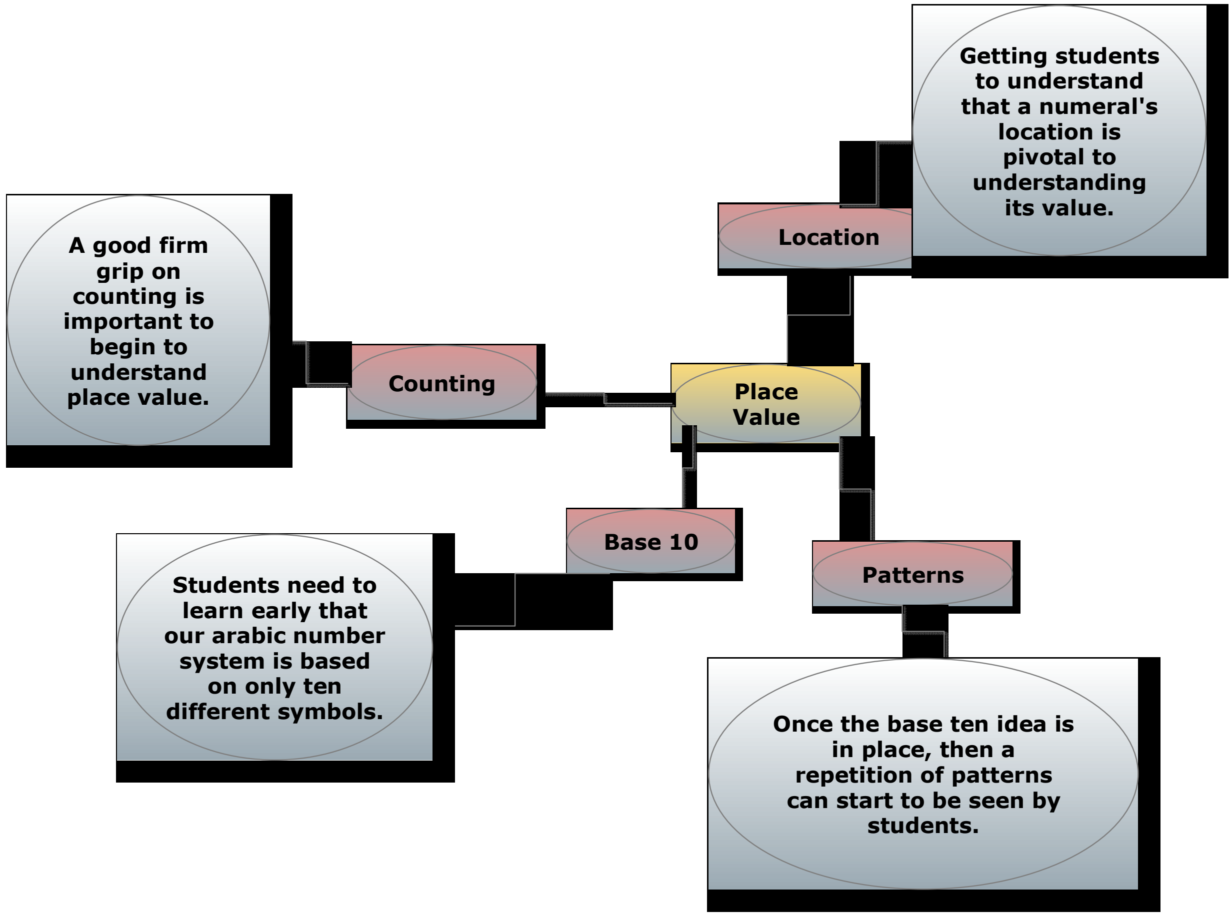


Figure 4.11. Liz’s Post-Orpda Concept Map

While the teachers were beginning to think about place value differently after Orpda, they still considered teaching as being one of the most important categories related to place value in both their pre- and post-Orpda concept maps. The largest number of teachers' highest main diagonal entries occurred in the teaching category in both their pre- and post-Orpda concept maps. This is not surprising since this was a methods course and many of the teachers were getting ready to begin their yearlong internship which would be their first experience at teaching. Discussions about how to teach mathematics were central throughout the entire class.

After focusing on the entries on the main diagonal of the squared adjacency matrices for each teacher, I then began to consider the entries below the main diagonal of the squared adjacency matrices. I considered how many non-zero entries were located below the main diagonal in each teacher's squared adjacency matrix. Non-zero entries below the main diagonal represent how many indirect connections teachers held between any two concepts. Specifically, I was interested in how many of these entries were greater than one which indicated that a teacher held a more connected understanding of place value.

Table 4.6 on the following page reveals these numbers from the teachers' pre- and post-Orpda concept maps. Not surprisingly, many of the teachers did not have entries below the main diagonal that were greater than one. This means that most of the teachers formed either direct connections between two different concepts or connected two different concepts using only one linking concept. However, we can see that five of teachers had entries greater than one in their pre-Orpda concept maps, with Cinderella and Lucy having entries of 20 and 16, respectively. This relates closely to the type of map the teachers drew. In the first phase of analysis, I noted that Cinderella and Lucy both chose to use a net structure for their pre-Orpda concept map. Since a net structured map is characterized by many connections among the included concepts, it

Table 4.6
Number of Entries Greater Than 1 Below Main Diagonal of Squared Adjacency Matrix

Teacher	**Pre-Orpda Concept Map**	**Post-Orpda Concept Map**
Scarlet	0	0
Betty	2	0
Tullula	0	0
Kate	0	0
Liz	0	0
Hank	2	1
Cinderella	20	4
Lucy	16	36
Tom	4	0
Lauren	0	0
Brad	0	15
Melissa	0	0

makes sense that Cinderella and Lucy had many entries with values greater than one below the main diagonal.

Four of the teachers had entries greater than one below the main diagonal in their post-Orpda concept maps. Interestingly, Brad had no entries greater than one below the main diagonal in his pre-Orpda concept map, but then showed 15 similar entries in his post-Orpda concept map. This reveals that he made many more indirect connections between different concepts related to place value in his post-Orpda concept map than he did in his pre-Orpda concept map. Cinderella, on the other hand, had only four entries greater than one below the main diagonal in her post-Orpda concept map compared with 20 similar entries in her pre-Orpda concept map. This reveals that Cinderella did not show as many indirect connections between the concepts related to place value after experiencing Orpda

Further analysis of Cinderella's pre-Orpda and post-Orpda concept maps shows that she included 10 different concepts related to place value in her pre-Orpda concept map and 21 different concepts in her post-Orpda concept map. Thus, looking at her entries below the main diagonal further shows that while she included more concepts in her post-Orpda concept map, she did not consider them as interconnected as the fewer concepts she included in her pre-Orpda concept map. The table also reveals that Lucy considered concepts related to place value in a very connected fashion as she had 16 entries greater than one below the main diagonal in her pre-Orpda concept map and increased that number to 36 in her post-Orpda concept map.

Summary

Analyzing data collected from the teacher's pre- and post-Orpda concept maps through multiple phases revealed several insights into how the teachers thought about place value both before and after Orpda. Table 4.7 found at the end of this summary reveals what was learned by analyzing each teacher's pre- and post-Orpda concept map across each of the three phases. The first phase of the analysis focused on the type of concept map teachers created before Orpda as compared to after Orpda. While most teachers seemed to connect the ideas related to place value using the same structure (net, chain, or spoke) both before and after Orpda, some teachers changed the way they perceived the connections between concepts after their work with Orpda. For some teachers, this change reflected a more conceptual understanding of place value after Orpda (as evidenced by the number of connections in the style of map), while, for one teacher, this change reflected a less conceptual understanding. Two of the teachers used a net style of concept map both before and after Orpda, revealing a deeper level of conceptual understanding in both cases.

The second phase of analysis focused on the categories teachers included in their concept maps both before and after Orpda. This phase revealed that, as a whole, the teachers were beginning to view place value from a more conceptual perspective after Orpda as compared to before. The categories of patterns, unitizing, and teaching all showed increased numbers of teachers that mentioned them in their post-Orpda concept maps. On the other hand, the number of teachers that included the category of types of numbers, a category considered to reflect a more procedural understanding, in their post-Orpda concept maps decreased.

The final phase of analysis used squared adjacency matrices created from each teacher's pre- and post-Orpda concept maps to better understand the connections teachers were making between the concepts they included in their maps. Examining the highest entries on the main diagonal of each teacher's squared adjacency matrix created from the pre-Orpda concept map revealed that most of the teachers viewed place value similarly before Orpda. The teachers' highest main diagonal entries were divided among only five different categories, therefore revealing that the teachers all considered many of the same categories as most important to understanding place value. After Orpda, the teachers had a more varied view of place value as their highest main diagonal entries from the squared adjacency matrices created from their post-Orpda concept maps appeared in all of the categories, except for two. This continued to suggest that teachers were beginning to recognize the categories that reflected a more conceptual understanding as being important to an overall understanding of place value after Orpda.

Entries that appeared in the squared adjacency matrices below the main diagonal indicated how many indirect connections existed between two different concepts in each teacher's concept map. Overall, analysis of these entries revealed that few of the teachers formed many indirect connections between different concepts both before and after Orpda. Teachers that created a net map showed the most indirect connections in their concept maps. Consequently, while the teachers seemed to be recognizing categories that reflected a more conceptual understanding of place value after Orpda, they were less inclined to connect all of these categories together in their map.

Table 4.7

Summary of Pre-Orpda and Post-Orpda Concept Map Analyses

Participant	**Category of Map Pre, Post**	**Topics Included Pre, Post**	**Concepts with Highest Number of Direct Connections (Main Diagonal) Pre, Post**	**Number of Indirect Connections (Below Main Diagonal) Pre, Post**
Liz	Chain, Spoke	Procedural, Conceptual	Types of Numbers – 6; Location, Base, Operations, Counting – all 2	0, 0
Lauren	Spoke, Spoke	Conceptual, Conceptual	Teaching – 9; Operations,Teaching- both 5	0, 0
Kate	Spoke, Spoke	Procedural, Conceptual	Position Names – 4; Unitizing – 3	0,0
Melissa	Spoke, Spoke	Procedural, Procedural	Operations – 5; Position Names – 7	0,0
Tullula	Spoke, Spoke	Procedural, Conceptual	Position Names – 8; Base – 3	0,0
Scarlet	Spoke, Spoke	Procedural, Conceptual	Teaching – 12; Teaching – 2	0,0
Cinderella	Net, Net	Procedural, Conceptual	Operations – 6; Symbols, Patterns – both 6	20,4
Joe	Spoke, None	Procedural, None	Types of Numbers – 14; None	0,None
Betty	Spoke, Spoke	Procedural, Conceptual	Position Names – 6; Symbols – 11	2,0
Hank	Net, Spoke	Conceptual, Conceptual	Base – 4; Position Names - 3	2,1
Tom	Chain, Chain	Procedural, Conceptual	Types of Numbers – 10; Teaching – 8	4,0
Brad	Spoke, Net	Conceptual, Conceptual	Operations – 6; Types of Numbers – 7	0,15
Lucy	Net, Net	Conceptual, Conceptual	Teaching – 6; Operations - 15	16,36

Question 3

What connections do teachers make between Orpda and the Arabic number system?

Throughout the week of Orpda activities, the teachers were encouraged to think about connections between the Orpda number system that they were learning and the Arabic number system that was already familiar to them, as their understanding of place value moved into the observing stage (Pirie & Kieren, 1994). One particular activity really forced the teachers to reflect on these connections and discuss what they had learned, and this activity was used as the focus for this analysis. Upon the completion of the first day of Orpda after the teachers were introduced to the language and symbols of the Orpda number system, they were given an @ skoobrat chart (written as @~~) to take home and fill out for homework. This assignment would be similar to asking elementary students to fill out a hundreds chart, counting up to the value of 200. A copy of a completed @ skoobrat chart can be found in Figure 4.3 on page 62. The entries in bold are the ones given to the teachers in the blank @ skoobrat chart and the non-bolded entries were filled in by the teachers.

At the beginning of the next class meeting, the completed @ skoobrat charts were collected and copies were made for analysis after class. The charts were then returned to the teachers to allow them to make changes as needed throughout the class discussion. After analyzing the teachers' answers, the first conclusion reached was that ten of the teachers filled out the chart correctly. Interestingly, of the remaining three teachers that had mistakes in their charts, all of the teachers made mistakes in the same region. They all correctly identified the symbol for star skoobrat (*~~). However, when trying to write the symbol for skoobrat star, two of the teachers, Melissa and Kate, indicated that this should be expressed as ~~*. Note that this is the reverse of the symbolic representation for star skoobrat. This answer made sense due to the fact that the words in the number name, skoobrat star, are also reversed. The remaining teacher, Liz, expressed skoobrat star as **, not recognizing that this is the same as the symbolic representation for doozle that appeared much earlier in the chart. All three of the teachers recovered at some point later in the chart and then completed the chart correctly from that point.

In order to understand what the teachers were thinking as they completed the @ skoobrat chart as well as force them to consider connections between the two number systems, all of the teachers were asked to respond to private blog questions. The questions were set up in a private blog format so that the teachers could not see other teachers' responses in hopes that this would force them to think about the connections on their own. The teachers were asked to respond to two different questions upon completion of their @ skoobrat chart. The two questions are given below.

1. Look for patterns in the symbols in the Orpda @ skoobrat chart. How do these patterns compare to the patterns found in a one hundreds chart that uses Arabic numerals?

2. Look for patterns in the number names in the Orpda skoobrat chart. How do these patterns compare to the patterns found in the hundreds chart number names?

After analyzing each teacher's responses to the two questions, it was apparent that all of the teachers noticed two similarities in the symbols between the @ skoobrat chart and a typical hundreds chart. Betty's response, quoted below, is reflective of many of the teachers' responses.

> Betty: Patterns can be found in each column and row of the orpda @ skoobrat chart and in the one hundreds chart. In each row, the number starts with the same symbol such as *, @, #, ^ in the orpda system and 1,2,3,4,5, etc. in the Arabic system until it reaches a repeat. The number is then preceded by a * or 1 to indicate skoobrat or hundreds respectively. In the columns, each ending symbol is the same such as *, @, #, ^, or ~ to represent the 'stars' place and 1,2,3,4,5, etc. to represent the ones place in the hundreds chart.

While it was encouraging to see that all of the teachers made this immediate connection between the charts for the two number systems, only five of the teachers made reference to the difference in the bases of the two number systems; that is, Orpda is a base 5 number system while the Arabic number system is base 10. In addition, two of the teachers initially believed Orpda to be a base 4 system. By not recognizing what it means for Orpda to be a base 5 number system as compared to the Arabic number system as base 10, some of the teachers had a difficult time finding deeper connections between the two number systems. Lauren's comment is evidence of this confusion.

> I do also know that the value of a *~ is equal to our 5, but it confuses me because it has 2 digits (which makes me think it should be 10).

Lauren's comment and many others similar to hers leads to the conclusion that some of the teachers were not considering the ideas of place value at this point as they had not recognized that Orpda is a base 5 number system.

The teachers were also asked to think about language connections that the two number systems shared. Again, a large portion of the teachers found some similar language connections. For instance, eight of the teachers noticed that the number names of atty, poundy, and carety in the Orpda number system were similar to the number names of twenty, thirty, and forty in the Arabic number system. In addition, eleven of the teachers recognized similarities between the names of the place value positions. They noted that the "stars" place in Orpda was similar to the "ones" place in the Arabic number system, while the "flubs" place was similar to the "tens" place and the "skoobrats" place was similar to the "hundreds" place.

However, some of the more implicit language connections between the two number systems were only noticed by a few of the teachers. Two of the thirteen teachers made reference to these connections in their responses. Tullula's response provided below was particularly reflective of how much it helped her to recognize these language connections.

> The first thing that I noticed when looking at the number names in the Orpda skoobrat chart was that I couldn't figure out how the names doozle, sholt, pouflube,carflube, and atty had anything to do with the numbers and symbols we were first introduced to (*, @, #, ^). However, I realized that atty came from @ and that atty is represented as @~. So that made sense to me, but doozle and sholt were still mysteries to me. Then it hit me…doozle and sholt are similar to eleven and twelve in the Arabic number system because the names of the numbers do not really relate to the others. Also, pouflube and carflube remind me of the teens…pou is # and car is ^, and the flube in each is flub. So that is just like thirteen, where the thir is three and the teen is ten.

After spending some time discussing these language connections during the next class meeting, several of the teachers noted how much these connections helped them to better understand Orpda. Lucy's comment provided below is reflective of what many of the teachers mentioned as follow up responses to their initial answers to the private blog questions.

> I now see that doozle and sholt are like the Arabic eleven and twelve. These were real stumbling blocks for me as we learned Orpda and I imagine they are for students, as well. Once you get to pouflube it's easier because you are combining a pound and a flub, like you would in Arabic for thirteen, which combines a three with a ten.

Summary

Upon completion of the first day of Orpda, the teachers were asked to complete the @ skoobrat chart and think about the connections in the symbols and language between Orpda and the Arabic number systems. Initially, many teachers recognized immediate connections in the symbols between the two charts at the end of each row and column. In addition, the teachers noticed how the names of the place value positions corresponded between the two number systems. However, some of the more indirect connections between the two number systems took the teachers some additional time to notice. Among those teachers that did notice these connections more quickly, their comments reflected ways in which they were able to draw upon their knowledge of the Arabic number system to better understand place value as it relates to the Orpda number system.

Chapter V

Conclusions and Recommendations

In this study, I attempted to begin to understand the relationship between Orpda and teachers' conceptual understanding of place value. The concepts surrounding place value are complex and abstract to children who are just being introduced to the base-ten number system. Consequently, teachers must recognize these complexities and provide various opportunities for children to develop a deep understanding of place value in order for them to be successful in future mathematics. This study indicates that Orpda and the reform methods used with the participants in the mathematics education course examined in the study do encourage reflection regarding these complexities.

The previous chapter reported results gleaned from the various data sources collected during this study, including the teachers' reflections on the Orpda activities, their discussions both in and out of the classroom, and their pre- and post-Orpda concept maps that illustrated their understanding of place value. Analysis of the teachers' "Aha!" moments from the various Orpda activities they participated in throughout the week of Orpda revealed that the teachers recognized the importance of unitizing, regrouping, and location as central components to developing a deeper understanding of place value. These three components have also been identified in the literature as central to understanding place value (Jones et. al, 1996). In addition, the teachers realized how the use of many different activities related to place value helped them strengthen their understanding of the Orpda number system.

Just as the analysis of the teachers' reflections from the activities revealed how the teachers were beginning to develop a more conceptual understanding of place value after Orpda, analysis of their pre- and post-Orpda concept maps further strengthened this conclusion. The teachers replaced concepts such as operations and types of numbers with concepts such as patterns and symbols in their post-Orpda concept maps. Also, analyzing the squared adjacency matrices created from each teacher's pre- and post-Orpda concept maps showed that the teachers maintained a belief that concepts related to teaching place value were just as important before Orpda as compared to after Orpda.

In order to understand and work with the Orpda number system, teachers had to draw on their prior knowledge of the base-ten number system and make connections between the two number systems. The final research question in chapter four presented conclusions drawn from analyzing answers that teachers provided to private blog questions that asked them to consider patterns in the symbols and language found in the Orpda and Arabic number systems. Teachers recognized common patterns in the symbols found in each row and column of the @ skoobrat chart and the one hundreds chart. Furthermore, teachers realized similar language patterns between the two number systems.

In this chapter, I present several claims that can be made from this study as a whole regarding teachers' understanding about place value and their thoughts about teaching mathematics. Each claim is supported by at least two data sources as shown in Table 5.1 below.

Table 5.1
Data Sources Supporting Claims

Conclusion	**Classroom Observations**	**Teacher Reflections**	**Concept Maps (Topics Included)**	**Concept Maps (Squared Adjacency Matrix)**
Orpda increased teachers' attention to the importance of unitizing to place value.		X	X	
Orpda encouraged teachers to reflect deeply on their teaching.	X	X	X	X
Concept maps show promise for revealing and documenting changes in conceptual understanding.			X	X
Orpda increased teachers' attention to the importance of patterns in understanding place value.	X	X	X	

In addition, recommendations for further research related to teachers' understanding about place value will be provided, as well as recommendations for practice.

Orpda leads teachers to understand the relationship that unitizing plays in developing a conceptual understanding of place value.

The act of unitizing involves combining multiple objects together to form a single entity and then being able to view the objects as a single entity or as individual objects. For example, combining 10 items to form a group of ten and seeing this as one ten or as 10 individual units. Research related to understanding children's concepts of ten reveals that many lack the ability to unitize (Baroody, 1990; Cobb & Wheatley, 1988). Consequently, these children struggle to understand place value as they do not recognize that ten objects can be grouped together to form the next grouping size in a numeral.

Orpda was created to help teachers recognize the importance of being able to view ten as a group of individual units, but also a single entity. Recognizing the importance of unitizing to developing a deeper understanding of place value, much of the week of Orpda was focused on helping the teachers better understand what unitizing means as well as its significance with place value. Analysis from both teacher reflections and from post-Orpda concept maps shows that Orpda did impact teachers' understanding of the concept of unitizing and that they recognized its importance to place value.

Among the 72 "Aha!" moments that the teachers provided after Orpda activities completed throughout the week, 27 of their reflections, from 11 of the 13 teachers, were related to unitizing. These reflections made reference to how much easier the Orpda number system became for the teachers when they were able to think in groups of flub, the base of the Orpda number system. The following quotes from Liz and Lucy are similar to other reflections provided by the teachers related to unitizing.

> Liz: Grouping using the base *~ helped me to think in Orpda and made memorizing the number symbols less important. (How Many? Activity)
>
> Lucy: Once I recognized the grouping by flubs, I started to think in flubs. (Fill the Flub Frames Activity)

The number of teachers who included unitizing in their concept maps related to place value increased from 2 in the pre-Orpda concept maps to 6 in the post-Orpda concept maps. While the reflections were written during the week of Orpda—and might reflect the assignment rather than conceptual understanding—it is significant to note that the post-Orpda concept maps were not assigned until the end of the course. In between the end of the Orpda experiences and the creation of the post-Orpda concept maps, the teachers were engaged in various other topics, including geometry, fractions, probability, and algebra. The increased presence of unitizing in the post-Orpda concept maps, even after a four-week break from place value, suggests that this does, indeed, represent conceptual change on the part of those who included unitizing at the end of the course.

<u>The experiences the teachers had with Orpda encouraged the teachers to reflect on their own teaching.</u>

Analysis of the topics included in the post-Orpda concept maps indicated that more teachers were reflecting on teaching than had been the case before Orpda, and the squaring of the adjacency matrices indicates how important they believed thinking about teaching to be. In addition, data from observations and from the teachers' reflections demonstrate thinking about and, in some cases, shifting perspectives on teaching place value.

Analysis of the teachers' pre- and post-Orpda concept maps revealed that one-third of the teachers included concepts related to teaching place value in their pre-Orpda concept maps in comparison with two-thirds of the teachers mentioning these concepts in their post-Orpda concept maps. In addition, teaching was the category that had the largest number of teachers with the highest entries on the main diagonals of the squared adjacency matrices, both before and after Orpda. This reveals that the teachers considered concepts related to teaching place value important to place value as a whole before Orpda, and maintained that belief after Orpda.

Classroom observations also support this claim. At the beginning of each day during the week of Orpda, the instructor placed various manipulatives in the middle of each table for the teachers to use to help them create images to represent problems. While she did not require the teachers to use the manipulatives, she encouraged their use as the teachers needed them.

In the beginning of the week, the teachers were reluctant to use the manipulatives. Towards the end of the week, the teachers were asked to solve word problems that involved Orpda language and notation as well as solve addition grids that used Orpda notation. After this class meeting, the teachers were asked to reflect on what helped them work through these problems. A particularly revealing discussion regarding the use of manipulatives developed between several of the teachers. Brad, Hank, and Lucy are all high school or college teachers while Kate is an elementary teacher.

> Brad: What helped me most…was to put out the blocks representing the sum, then remove the one quantity I knew to see what was left.
>
> Kate: I like the way you solved your problems with the manipulatives. That is how I solved the word problems, but it didn't occur to me to even use the manipulatives with the addition grids. If you don't mind me asking, what made you use manipulatives in that activity? For some reason, I thought that teachers of upper grades were "anti-manipulative."

Hank: Curses! I can't speak for everyone, but I am not necessarily "anti-manipulatives." The problem is I have been through so many years of math without them that I just don't think about them. That's my goal through this coursework, to get familiar using tools other than pencil and paper.

Lucy: I thought the Orpda blocks were so helpful. When I tried to do the addition without them it was much harder.

This discussion reveals how Orpda challenged some of the teachers' thoughts about how mathematics is learned and taught. Just as Hank noted, many teachers are not accustomed to using manipulatives mostly because they did not use them much as they learned mathematics for themselves. However, after using them some of the teachers recognized the benefits of being able to create images to represent situations and see how the mathematics works in each problem.

At the end of the week, the teachers were asked to reflect on the week of Orpda as a whole and provide both positive and negative feedback on their thoughts regarding the use of Orpda to think about place value. All of the teachers made reference to how Orpda encouraged them to think about their own teaching practices. In particular, the teachers mentioned how much discovering the mathematics for themselves helped them to better understand place value. To summarize, some of the teachers' quotes are included below.

Betty: Previously, I tended to lean towards very structured activities with much modeling from the instructor. I now think that student discovery is an essential part of learning.

Hank: One of the biggest challenges…is the ability to open my mind to other ways of thinking and learning. I don't naturally think about hands-on methods, so I really wanted to figure out how to use all those different methods. I tried to arrange the manipulatives several different ways that I thought students might do and think about different possibilities.

Cinderella: This week reinforced my belief in students needing different and multiple exposures of concepts to fully understand. Doing the different tasks helped me understand Orpda better.

Lauren: I did not realize how important the place value concept was until I started doing activities with Orpda, but it made me realize just how much emphasis I will need to put on it in my own classroom.

<u>Concept maps show promise for revealing and documenting changes in conceptual understanding.</u>

Concept maps were a primary source of data used in this study to understand and reveal changes in how the teachers thought about place value concepts both before and after Orpda. The first phase of analysis involved classifying each teacher's pre- and post-Orpda concept map as a net, spoke, or chain map, categories developed by Kinchin and Hay (2000). After doing so, I was able to recognize where changes in conceptual understanding occurred among the different teachers in the study. Liz and Brad showed more conceptual understanding as they created a different style of concept map after Orpda that was considered to represent a deeper level of conceptual understanding. On the other hand, Hank, showed less conceptual understanding as he changed from a net style concept map before Orpda to a map that followed a spoke style after Orpda.

Borrowing from the area of graph theory, adjacency matrices provided one way to begin to understand connections teachers made between the various concepts that they included in their pre- and post-Orpda concept maps. Furthermore, following the technique developed by Lapp, Nyman, and Berry (2010) of examining squared adjacency matrices provided another level of analysis for the teachers' concept maps. Analyzing the entries on the main diagonals of each teacher's squared adjacency matrix created from the pre- and post-Orpda concept map revealed that many of the teachers created post-Orpda concept maps that reflected a more conceptual understanding of place value. This was evident by the fact that categories such as operations and types of numbers, which are considered to reflect a more procedural understanding of place value, contained the largest number of teachers with the highest entries on the main diagonals. After Orpda, all of the categories, except two, were among the teachers' highest entries along the main diagonal. These categories included base, symbols, and patterns, among others, which are recognized as categories that reflect a conceptual understanding of place value.

Entries below the main diagonal of the squared adjacency matrices revealed how many indirect connections teachers included between two different concepts in their pre- and post-Orpda concept maps. Since conceptual understanding is defined as being rich in connections between concepts, this was an important part of the analysis used to document any changes in conceptual understanding among the teachers. While forming each teacher's adjacency matrix as well as squaring all of the matrices was a time-consuming and tedious process, the entries below the main diagonal made recognizing the indirect connections much easier for each teacher as compared to just looking at the map alone.

All three phases of analysis of the teachers' pre- and post-Orpda concept map worked together to illustrate a picture of each teacher's changes in conceptual understanding throughout the study. For example, phase one of the analysis revealed that Cinderella constructed a net style map in both her pre- and post-Orpda concept maps. This suggested that she maintained a high level of conceptual understanding throughout the entire study. However, analysis of the entries below the main diagonal revealed that the number of non-zero entries dropped from 20 to 4. Consequently, while she still constructed her concept map using a net style after Orpda, she did not use as many connections between different concepts in her map as she did in her pre-Orpda concept map. Therefore, I can conclude that Cinderella still viewed the concepts related to place value as interconnected after Orpda, but not as strongly as she did before Orpda. Without considering all three phases of analysis of the concept maps, I would not have been able to gain as deep of an understanding of the development of conceptual understanding for each teacher throughout the study.

Orpda increased teachers' attention to the importance of patterns in understanding place value.

Throughout the entire week of Orpda, the teachers were continually asked to think about patterns and connections that they noticed between the base-ten and Orpda number systems. For example, on Monday, the first day of Orpda, after the teachers were introduced to the word names for numbers in the Orpda number system, the instructor asked the teachers to think about any patterns that they noticed between Orpda and base-ten. Hank recognized patterns in the word names for each additional group of the base, such as atty, poundy, and carety, that were similar to the word names of twenty, thirty, and forty in the Arabic numerals. Tom also saw patterns in how the word names for the teens, such as thirteen, are constructed, which were similar to the construction of the word names of pouflube and carflube in the Orpda number system.

The teachers' homework assignment for Monday evening asked them to continue to recognize patterns as they completed their @ skoobrat charts, and many of the teachers saw similar patterns between the symbols and language of both number systems. In addition, analysis of the teachers' pre- and post-Orpda concept maps revealed the importance of patterns to understanding place value. None of the teachers included concepts related to patterns in their pre-Orpda concept maps, while seven of the teachers included the idea of patterns in their post-concept maps. Furthermore, the teachers' reflections on the activities and the week as a whole revealed how much recognizing patterns helped them work with Orpda, as documented by the teachers' quotes provided below.

> Lucy: I finally saw the pattern of putting the symbols together. (Counting Bags Activity)
>
> Tom: What helped me was using the flubs to create a pattern. I worked off of the flubs to create a new combination. (How Many Ways Activity)
>
> Cinderella: The chart [@ skoobrat] is really great to see the patterns in Orpda. (End of the Week Reflection)

Recommendations for Further Research

This study was conducted using a small number of teachers that were enrolled in a five-week summer course, taken as an elective to meet requirements to complete their degree at the university. Orpda was the focus of the course for one of the five weeks that the teachers met. During this time, data was collected from numerous data sources and each source was then analyzed in order to understand the teachers' relationships between Orpda and their understanding of place value. In order to strengthen the claims from this study, additional studies need to be conducted using different groups and numbers of teachers, as well as times frames for using Orpda. Furthermore, data collected from the use of Orpda in other settings, such as professional development meetings, with teachers that have various levels of teaching experience would also help to support the conclusions from this study.

Finally, the use of follow-up interviews to further enhance the use of concept maps is recommended for future research. This study employed the use of concept maps as a means of assessing the teachers' conceptual understanding of place value. Pre- and post-Orpda concept maps were analyzed across three stages in order to gain the most understanding of the teacher's thinking. The first phase of the analysis involved classifying the teachers' pre- and post-Orpda concept maps as net, spoke, or chain structures as defined by Kinchin and Hay (2000). A map that followed a net structure was considered to reveal the deepest level of conceptual understanding as it was characterized by many connections between all of the concepts included in the map. Spoke and chain maps were considered to demonstrate less conceptual understanding as fewer connections existed between the concepts.

This phase of the analysis revealed that three teachers changed the structure of their post-Orpda concept map as compared to their pre-Orpda concept map. In particular, Hank constructed his pre-Orpda concept map in a net-type fashion and created a spoke structure for his post-Orpda concept map. This suggested that Hank's thinking about place value became less conceptual after Orpda as compared to before.

Throughout the week of Orpda, Hank was particularly reflective during the activities and class discussions. On the first day of Orpda when the teachers were discussing how to represent flub (*~) after being introduced to the first four numbers in the Orpda number system, Hank contributed to the discussion by explaining that the different place values in the symbolic representation of flub represented different grouping sizes of the base. As the week progressed, Hank caught on quickly to all of the activities that were done in Orpda and made a conscious effort to stay in Orpda rather than converting problems to the base-ten number system.

Hank exhibited a strong understanding of place value throughout the entire week of Orpda. Therefore, questions were raised as to why his post-Orpda concept map was structured in a way that indicated less conceptual understanding. Consequently, while this study does contribute to a line of research related to assessing conceptual understanding through the use of concept maps, Hank's situation suggests that accompanying interviews with the teachers would also be beneficial. If I had been able to interview Hank after he created his post-Orpda concept map, I would be able to better assess whether his conceptual understanding of place value did in fact decrease after Orpda as he would have been able to explain why he chose to include particular concepts and why he linked them together in the way that he did.

A follow-up interview with Brad, another high school teacher, would have also been helpful to understand a particular aspect of his post-Orpda concept map. Brad included the concept of converting between bases in his post-Orpda concept map, but did not do so in his pre-Orpda concept map. Seeing this led me to question whether Brad focused more on converting problems in Orpda to base-ten rather than staying in Orpda and focusing on place value. This is a concern when using alternate bases (McClain, 2003). However, the creators of the Orpda number system specifically chose to use symbols rather than numerals as well as unfamiliar language in order to try to lessen the temptation among teachers to convert to base-ten. While answering this question was not the focus for this particular study, a follow-up interview with Brad and other teachers could shed some light on how they worked with Orpda throughout the time of the study.

Recommendations for Practice

Orpda is an invented number system created with the hopes that it would challenge teachers to reconceptualize their understanding of topics related to place value, and the purpose of this study was to explore that relationship. Conclusions from this study indicate that Orpda does impact teachers' understanding of place value as topics such as unitizing and patterns were made more explicit after spending time working in Orpda. However, additional conclusions from this study reveal that Orpda also impacts the way teachers think about teaching mathematics in a more conceptual way.

Classroom environment that supports reflection. One recommendation for successfully helping preservice teachers manage both roles of teacher and student in a methods class, as indicated from this study, is to provide opportunities to allow the teachers to reflect and then foster a classroom environment in which everyone feels comfortable to openly share their thoughts and reflections in order to encourage meaningful discussions. During the week of Orpda, the teachers had to assume two roles – the role of a student and the role of a teacher. Since Orpda was new to them, they first had to take on the role of a student as they worked through the various activities to gain an understanding of Orpda for themselves. This required them to "fold back" to their experiences of working with the base-ten number system, draw on their prior knowledge of place value, and apply it to help them understand the Orpda number system (Pirie & Kieren, 1994). In order to encourage them to reflect on their own understanding, social norms for the class were put in place at the beginning of the week. The teachers were allowed to sit with the same group throughout the week so that they could feel more comfortable. In addition, the instructor had set up the classroom environment as one that fostered discussions through an inquiry-based approach, and the teachers felt comfortable sharing their thoughts and reactions as they thought about their own place value understanding.

At the same time, the teachers assumed the role of a teacher as they reflected on their own teaching through their work with the Orpda activities. Surprisingly, the teachers did not seem to struggle to find a balance between these two roles throughout the week. Part of that ease of transition was due to the many opportunities the instructor provided that allowed the teachers to reflect on their own thinking. Tom alluded to this fact in his end of the week reflection as he noted, "The reflection on my own metacognition and my students was invaluable for the fact that many concepts I teach might begin for them like Orpda was for me." One struggle in teaching methods classes arises as the instructor tries to find a way to allow preservice teachers to assume a dual role of student (in order to deepen their content knowledge) and teacher (in order to strengthen their pedagogical knowledge).

Choose tasks to provide appropriate challenge. Another challenge that arises when working with teachers in a methods class, particularly a mathematics methods class for elementary teachers, involves finding a way to make their knowledge about the subject more explicit so that the teachers are aware of the difficulties their students will experience when learning these topics for the first time. The teachers' familiarity with the topics, such as place value, can interfere with their realization of the difficulties their students will experience when first exposed to these new topics. Consequently, just as this study suggests, specific tasks must be chosen that provide the teachers opportunities to become students and struggle for themselves in the same way that their own students will struggle. The Orpda activities that the teachers participated in throughout the week were designed to meet both of these needs. The instructor and I purposefully selected activities that would make the teachers' knowledge about counting, unitizing, regrouping, and number relationships -- four key components identified in the literature as central to developing a conceptual understanding of place value (Jones et. al. ,1996) – explicit. In addition, we chose activities that would also allow the teachers to struggle in the same areas where their students would struggle.

Upon completion of the week of Orpda, many of the teachers' reflections about the week as a whole alluded to the struggles they experienced, and, consequently, the importance of choosing appropriate tasks for their own students to allow them to work through these same struggles. Examples of some of the teachers' reflections regarding their experiences are provided below.

Kate: I think this is a good lesson because it helps us to understand what our younger, or less developed, students are experiencing when trying to learn about our number system for the first time.

Lucy: This was a very eye opening experience for me. I struggled a lot in the beginning. The most powerful outcome of the week was just being humbled about learning something that felt so different.

Lauren: I was challenged this week to stretch my thinking to a whole new level with the Orpda system of numbers!

Choosing the appropriate task to both further the teachers' own understanding as well as encourage them to reflect on their own teaching is not easy. However, it is an extremely important part of teaching methods classes and must be done with care. Decisions about which tasks to choose must be informed by research and the goals that the teacher educator wants to achieve. Teachers must be given opportunities to understand that in order to develop conceptual understanding in their own students, they must possess the same understanding for themselves. As a result of choosing the appropriate tasks and providing meaningful experiences, teachers can then start to truly understand what it means to teach mathematics.

Summary

Finding a way to help teachers deepen their own understanding of the mathematics they teach, particularly at the elementary level, is a challenging task for mathematics teacher educators. The teachers' familiarity with the content prevents them from recognizing the struggles that their students will face when first introduced to new mathematical ideas. In addition, the teachers' own level of understanding may be masked by their familiarity with the content as their answers to questions have become automated responses and do not require a deep level of thinking (Ball, 1998).

In the area of place value, teachers' familiarity with the base-ten number system can prevent them from recognizing the importance of four key components – counting, unitizing, regrouping, and number relationships - necessary to developing a conceptual understanding of place value, as outlined by Jones et. al. (1996). Much of the research related to these four key components identifies the struggles children experience when attempting to learn about place value. In particular, inconsistencies in the language patterns inherent to the base-ten number system make memorizing the counting sequence difficult for children (Cotter, 2000; Ho & Fuson, 1998). Furthermore, children need numerous experiences with grouping objects in order to recognize that ten objects can be viewed as a single entity and develop the ability to unitize (Cobb & Wheatley, 1988). Teachers have had many experiences with the base-ten number system and, therefore, have a hard time identifying with these struggles of their students.

For this research, I used an instrumental case study design that considered teachers' experiences with Orpda as the case, and I employed qualitative approaches to analyze multiple data sources collected from teachers enrolled in a mathematics education graduate course. Upon conclusion of the data analysis, I found that the use of the Orpda number system was one way of helping teachers look at their own understanding of place value more deeply, as well as identify the struggles that come with attempting to learn about place value concepts. In addition, many of the teachers' reflections throughout the week also indicated that their experiences with Orpda helped them recognize the advantages of using manipulatives and working through numerous hands-on activities in order to rediscover the important ideas related to place value for themselves. While the Orpda number system would not be used with elementary students, the conclusions from this study suggest that Orpda is an effective method to help teachers deepen their own understanding of place value concepts. As a result of their experiences with Orpda, teachers will hopefully provide similar experiences in their own classrooms to allow their students to discover place value concepts. In addition, teachers will be able to empathize with the struggles their students are experiencing and understand ways to help students overcome these struggles and develop a deeper level of understanding related to this important area of mathematics.

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Appendices

Appendix A

Discussion Board Questions

Question	Form
Look for patterns in the symbols in the Orpda @ skoobrat chart. How do these patterns compare to the patterns found in a one hundreds chart that uses Arabic numerals.	Private Blog
Look for patterns in the number names in the Orpda @ skoobrat chart. How do these patterns compare to the patterns found in the hundreds chart number names?	Private Blog
Think about the Addition Grids worksheet that you did in class today. What helped you solve the problems? Where did you struggle? What are you still confused about?	Group Discussion
Consider the entire week of activities related to Orpda. Now think about Chapter 3 in your text – Knowing and Understanding Students as Learners. In what ways did your personal strengths, weaknesses, beliefs, and/or dispositions about teaching and learning mathematics (or about Orpda) enhance or inhibit your participation? Please be honest with your thoughts about Orpda to help us improve the unit. All constructive criticism supported with reasoning would be appreciated and will not affect your grade.	Group Discussion

Appendix B

Instructor Interview Questions

1. How would you describe the group of teachers as a whole? How do they compare with other groups of teachers where you have done Orpda?

2. When working in Orpda, what aspects related to place value did you try to emphasize to the teachers?

3. What other topics did you cover for the rest of the summer? Did you present these topics in a similar way to Orpda (i.e., working through activities, small groups, discussions, etc.)?

4. Were there any specific teachers that you remember being particularly reflective during Orpda and even beyond in the remaining weeks of the class?

5. Was there anything that happened during the remainder of the summer that might influence the results of the teachers' second concept maps about place value?

Appendix C

Teacher Demographic Sheet

Name:	**Phone:**

High School Mathematics Courses (Check each course completed with C or better)			
Algebra I	Calculus I	Statistics	Trigonometry
Algebra II	Pre-calculus	Geometry	Other:

College Courses (Check each course or their equivalent completed with C or better)	
MA 110 or Algebraic Reasoning	MA 201 or Number System
MA 115 or Statistical Reasoning	MA 202 or Probability, & Statistics
MA 123 or Finite Mathematics	Undergraduate math major
MA 125 or Basic Calculus	Other

Level of Mathematics Anxiety (Circle one)				
I LOVE math!	Low anxiety	Medium anxiety	High anxiety	Whoa!!! I gotta do math??

Desired grade level for teaching (check as many as apply)														
Pre-K	K	1	2	3	4	5	6	7	8	9	10	11	12	Higher Ed

Complete the following:

Mathematics is…

Describe the teacher's role in a mathematics classroom:

Describe the student's role in a mathematics classroom:

Appendix D

Codes for Discussion Board Questions and Classroom Observations

<u>Pirie and Kieren's Model for the Growth of Mathematical Understanding</u>

- Primitive knowing – prior knowledge teachers bring to the study
- Image making – using concrete objects and/or drawings to work with Orpda
- Image having – possessing a mental image of the quantities within the Orpda numeration system
- Property noticing – recognizing properties related to the Orpda numeration system
- Formalizing – extending the recognized properties to the entire numeration system
- Observing – reflections on what helped to increase understanding
- Structuring – thinking about observations as part of a theory
- Inventizing – possessing a full structured understanding and extending it to new situations
- Folding back – returning to a previous level of understanding to approach a new problem

<u>Key Components of Place Value Understanding</u>

- Counting – learning oral and written language associated with Orpda
- Unitizing – recognizing objects as singles and groups
- Regrouping – breaking down numbers to recognize different ways to create the same quantity
- Number relationships – recognizing the number relationships within the Orpda numeration system

<u>Experiences With Orpda</u>

- Frustration – struggles with grasping the Orpda numeration system
- Aha! moment – an experience that provided additional understanding
- Relationships to base ten – recognizing relationships within the Orpda numeration system that are inherent to the base-ten number system

Appendix E

Categories for Topics Included in Concept Maps and Their Definitions

- Location: references to the idea that the position of a digit in a number determines the digit's value
  Phrases coded this way include: The base determines the actual value of each digit; Value of places; Digit value; Number placement; The position of a single digit in a whole number or decimal number containing one of more digits; The value of a digit as determined by its position in a number

- Base: references to the idea that the Arabic number system is a base-ten number system
  Phrases coded this way include: Values assigned by the base; Actual values depend on a base; We use a base 10 system; Base ten; Bases;

- Unitizing: references to the ideas of grouping by tens and anchoring to ten
  Phrases coded this way include: How many of each group; Grouping; Making groups of ten units; Groups of 10 bundles; Value of 10; Anchor to tens

- Relationships: references to the ten-to-one ratio between consecutive place values in a multi-digit number, mentions using place values to compare and order numbers, relates using place value ideas in real-world situations
  Phrases coded this way include: Decimal indicates point between 0 and -1 powers; Part/whole relationships; Decrease by a factor of 10; Relationships between numbers; Comparing; Ordering

- Symbols: references to symbols used in the base-ten number system
  Phrases coded this way include: Digits-often 0-9 and A-Z; Values of digits (0,1,2,…); Digits 1-9; Symbol; 0,1,2,3,4,5,6,7,8,9;

- Patterns: references to the fact that the Arabic number system is patterned
  Phrases coded this way include: Simple pattern; Patterns; Base '10' patterns; Patterned

- Operations: references to mathematical operations such as addition, subtraction, multiplication, and division used for calculations
  Phrases coded this way include: Addition; Subtraction; Multiplication; Division; Operations; Algorithms

- Zero as placeholder: references to the importance of recognizing the use of zero as a placeholder in multi-digit numbers when no groups of a certain size are present within the quantity
  Phrases coded this way include: Place holder; Concept of zero; Importance of zero; Recognizing zero;

- Position Names: references to the different names of positions within a multi-digit number
  Phrases coded this way include: Units, tens, hundreds; One's place; Ten's place; "Places"; Thousands; Ten thousands; Tenths; Hundreths; Thousandths; Ten thousandths;

- Counting: references to counting as an important component to learning a number system
  Phrases coded this way include: Counting; One-to-one correspondence;

- Types of numbers: references to various types of numbers within the real and complex number system, including whole numbers, integers, rational numbers, such as decimals and fractions
  Phrases coded this way include: Fractions or decimals; Rational; Irrational; Non-repeating decimal; Repeating decimal; Mixed numbers; Whole numbers; Real numbers; Integers; Natural numbers; Percents

- Teaching: references to ideas related to teaching place value, including mention of the NCTM standards, using manipulatives, completing activities, and on-going assessment of place value understanding
  Phrases coded this way include: Manipulatives and games; Math content; NCTM standards and content standards; Flats, rods, cubes; Base-ten blocks; Discovery; Repetition/drill; Assessing; Learned in elementary school; Better when taught using exploration activities rather than just simple instruction

- Converting between bases: references to being able to convert between bases of different number systems
  Phrases coded this way include: Converting between bases

Vita

Jamie Howard Price was born on June 20, 1980 in Louisburg, North Carolina. She graduated from Louisburg High School in 1998 and entered North Carolina State University in Raleigh, North Carolina in August 1998 to pursue a degree in engineering. Realizing her love for mathematics, she changed her major to mathematics and transferred to East Tennessee State University in Johnson City, Tennessee in August 1999 and completed her Bachelor of Science degree in mathematics in May 2002. Upon completion of her undergraduate degree, she remained at East Tennessee State University and completed her Masters of Science degree in May 2004. Research for her thesis was conducted in the area of graph theory.

After finishing her graduate degree, Jamie moved to Port Saint Lucie, Florida and taught mathematics at Indian River Community College for two years. During this time, she taught various courses, ranging from College Algebra to Calculus 2. Recognizing that she had a passion for teaching mathematics, she moved back to Johnson City, Tennessee in August 2006 and assumed her current teaching position at East Tennessee State University. In August 2007, she enrolled in the doctorate program in Education with an emphasis in Math Education and will complete her degree in May 2011. Upon completion of her doctorate degree, Jamie will continue teaching at the post-secondary level, teaching various courses in both mathematics and mathematics education.